Appendix A

We shall designate two different event reference frames S and S', with two observers one in each reference frame. Joe ham sees the event (dB field) in his reference frame S. The other observer would see the event (dE field) in their reference frame S'. The S' reference frame moves with speed v with respect to frame S. The time is t = 0 when the origin's of both O and O' are at the same starting point, and the event occurs at some later t. Joe ham in S describes events at coordinates (x,y,z,t) and the other observer describes events in their reference frame as (x',y',z',t').

Using the concept of free space moving charge, presented in "Why Antennas Radiate" (equation 3, with $\theta = 90°$) Joe ham would observe the B field moving at a velocity v to be:

1) $$B = \frac{v}{c^2}\gamma E'$$

Where $\theta = 90°$, the angle between the B field and the wire. The B field velocity v is already in the S reference frame.

Taking the time derivative of the B event in S, to both sides of the equation:

2) $$\frac{dB}{dt} = \frac{d}{dt}\frac{v}{c^2}\gamma E'$$

Next using the product rule:

3) $$\frac{dB}{dt} = \frac{\gamma E'}{c^2}\frac{dv}{dt} + \frac{v}{c^2}\frac{d\gamma E'}{dt}$$

Assuming the velocity is constant (no acceleration), the first term in equation 3 goes away:

4) $$\frac{dB}{dt} = \frac{\gamma E'}{c^2} * 0 + \frac{v}{c^2}\frac{d\gamma E}{dt} = \frac{v}{c^2}\frac{d\gamma E'}{dt}$$

5) $$\frac{dB}{dt} = \frac{v}{c^2}\frac{d\gamma E'}{dt}$$

We could utilize the binomial expansion of $(1+x)^r$ for the gamma term, if we wanted to, where:

6) $$(1+x)^r = 1 + rx + \frac{r(r-1)}{2!}x^2 + \frac{r(r-1)(r-2)}{3!}x^3 + \frac{r(r-1)(r-2)(r-3)}{4!}x^4 + \ldots$$

Where $x = -\frac{v^2}{c^2}$ and r = -½ so that:

7) $$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \cong 1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \frac{5v^6}{16c^6} + \frac{35v^8}{128c^8} + \ldots$$

Since $v^2 << c^2$ the second and remaining terms would go to zero, therefore we would arrive very close to $\gamma = 1$. For the present time however, we will leave in the gamma term.

Substituting into equation 5 for the *E'* field in terms of static charge (*E*' = q/4πεr²) in S' we get:

8) $$\frac{dB}{dt} = \frac{v}{c^2}\frac{d}{dt}\left[\frac{\gamma q}{4\pi\varepsilon_0 r'^2}\right]$$

Moving *dt*, on the left side of the equation, over to the right side of the equation we can combine *v* and *dt* to get *dl=vdt,* since *v* and *dt* are both in S.

9) $$dB = \frac{1}{c^2}\frac{d}{dt}\left[\frac{\gamma q}{4\pi\varepsilon_0 r'^2}\right]dl$$

We know in the S' reference frame that *dq* = *nqAdl'* . But *dl'*= *γdl*. Where n is the number of charges in the differential segment volume *Adl' or dV'*. We can move all of *dq* or *nqAdl'* to the S reference frame by making a substitution and multiplying by gamma, d*q* = *γnqAdl*:

10) $$dB = \frac{1}{\varepsilon_0 c^2}\frac{\gamma}{dt}\frac{nqAdl}{4\pi r^2}dl$$

But $dl = v_d dt$ *so*:

11) $$dB = \frac{1}{\varepsilon_0 c^2}\frac{\gamma}{dt}\frac{nqAvdt}{4\pi r^2}dl$$

Now the *dts* cancel since they are in S. Additionally knowing that:

$$\mu_0 = \frac{1}{\varepsilon_0 c^2}$$

We get:

12) $$dB = \frac{\mu_0 \gamma nqAv_d}{4\pi r^2} dl$$

Lastly knowing that $I = nqAv_d$, we arrive at:

13) $$dB = \frac{\gamma \mu_0 \mathrm{Idl}}{4\pi r'^2}$$

Now if we wanted to use the binomial approximation in equation 7:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \cong 1 + \frac{v^2}{2c^2} + \frac{3v^4}{8c^4} + \frac{5v^6}{16c^6} + \frac{35v^8}{128c^8} + \ldots$$

14) $$dB = \frac{\mu_0 \mathrm{Idl}}{4\pi r'^2}\left[1 + \frac{v^2}{2c^2}\right]$$

But v<<c so we end up with the magnitude form of the Biot-Savart law, with $\theta = 90°$:

15) $$dB = \frac{\mu_0 \mathrm{Idl}}{4\pi r^2}$$

Note: Many Thanks to Dan Creveling for pointing out *dl'= γdl not γdl'= dl*