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Design of a Two-band Loaded Dipole Antenna

Calculate the LC trap values given the physical size of the antenna and two desired resonant frequencies.

I wanted to put up a dipole antenna in my attic but didn't have space for a full sized 40 meter antenna. I came across a *QST* article[1] by Luiz Lopes, CT1EOJ, on a method to calculate the values of loading coils to resonate a short antenna on a frequency lower than the natural resonant frequency. It dawned on me that if adding inductance would effectively lengthen an antenna then adding capacitance would effectively shorten it and that Lopes' method would work to find the capacitance as well.

A parallel *LC* circuit is inductively reactive below its resonant frequency, and a capacitively reactive above its resonant frequency. So if I replaced Lopes' loading coil with a parallel *LC* trap. I could find *L* and *C* values that would make the trap have just the right inductive reactance at one frequency to resonate an antenna at that frequency, and at the same time have the right capacitive reactance to make the antenna effectively shorter, and hence resonant, at some higher frequency. This article explains how to calculate the *LC* values given the physical size of the antenna and the two desired resonant frequencies.

Calculating the Reactances

We can compute the reactances needed at two specified frequencies using the method of Lopes. When we shorten the antenna we are removing a piece of the full size dipole and replacing the cut piece with an inductance. Figure 1 shows half of a dipole where the dashed part represents the piece removed to shorten the antenna. Only half of the dipole is shown since the locations of the loading elements will be symmetrically placed about the center. These calculations also apply to a loaded vertical antenna. The location of the piece and its length are both design choices. One sets the total length by some external constraint, then chooses the location of the cut to optimize some aspect of the antenna behavior. The needed inductive reactance value is given by the difference in reactance between points "A" and "B" in Figure 1. Note that if the antenna is too long it is the same as adding a negative gap to the normal dipole length.

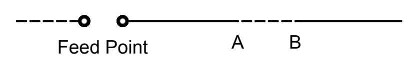

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Figure 1 — One side of a symmetrical loaded dipole shows the gap between A and B where length is removed and a trap is inserted.

Lopes models the antenna as a single-wire transmission line above ground. The reactance at any point along the transmission line is given by the transmission line equation, $Z = jZ_0 \tan(\beta)$, where Z_0 is the characteristic impedance of the transmission line and β is the distance in electrical degrees from the center of the antenna to some point on the antenna. β is between 0 and 90 degrees as we move out to a quarter wavelength along the dipole arm. For a single-wire transmission line above ground the characteristic impedance is,

$$Z_0 = 138 \log_{10}\left(\frac{4h}{d}\right)$$

where h is the height of the antenna above ground, and d is the diameter of the wire, both in the same dimensions.

Note that the placement of the trap is governed by the requirement that the inner length must be shorter than a quarter wavelength at the higher frequency. Given an antenna length and two frequencies at which we would like it to be resonant, we can use the method of Lopes to calculate the value of inductive reactance for the lower frequency and the value of the capacitive reactance at the higher frequency. These are the effective values, and we need a trap that would have these reactances at the two frequencies.

Parallel LC Network

The reactance of a parallel *LC* circuit is,

$$X(\omega) = \frac{-\omega L}{1 - \left(\frac{\omega}{\omega_0}\right)^2} \tag{1}$$

where ω_0 is the resonant frequency of the circuit. It is more convenient for our purposes to re-write this equation in terms of ω_0 and X_0 the magnitude of the reactance of either of the components at the resonant frequency. With some algebraic manipulation we get,

$$X(\omega) = \frac{-X_0}{\left(\frac{\omega_0}{\omega}\right) - \left(\frac{\omega}{\omega_0}\right)} \tag{2}$$

where $X(\omega)$ is the effective reactance of the trap at frequency ω.

We want the trap to have reactance X_1 at the lower frequency ω_1, and reactance X_2 at the higher frequency ω_2. From equation (2),

$$X_1 = \frac{-X_0}{\left(\frac{\omega_0}{\omega_1}\right) - \left(\frac{\omega_1}{\omega_0}\right)} \quad (3)$$

and

$$X_2 = \frac{-X_0}{\left(\frac{\omega_0}{\omega_2}\right) - \left(\frac{\omega_2}{\omega_0}\right)} \quad (4)$$

Divide equation (4) by (3) to eliminate X_0 to get an equation in terms of the reactance ratio at the two frequencies,

$$\frac{X_2}{X_1} = \frac{\frac{\omega_0^2}{\omega_1} - \omega_1}{\frac{\omega_0^2}{\omega_2} - \omega_2}$$

Since we know X_2 and X_1 we can solve this last equation for ω_0^2,

$$\omega_0^2 = \frac{\omega_1^2\omega_2 - \frac{X_2}{X_1}\omega_2^2\omega_1}{\omega_2 - \frac{X_2}{X_1}\omega_1}$$

We use this to solve for X_0,

$$X_0 = X_1\left(\frac{\omega_1}{\omega_0} - \frac{\omega_0}{\omega_1}\right) = X_2\left(\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2}\right).$$

The *L* and *C* Values

Given the antenna length that one wants to use, and the desired two resonant frequencies, we can calculate ω_0^2 and X_0. From these we calculate the values of *L* and *C* that comprise the trap,

$$X_0 = \omega_0 L = \frac{1}{\omega_0 C} \text{ and } \omega_0^2 = \frac{1}{\sqrt{LC}}$$

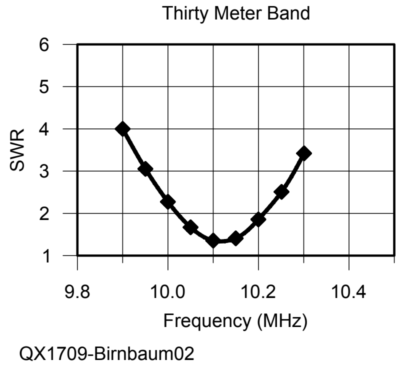

Figure 2 — SWR calculated using NEC for the 30 m band.

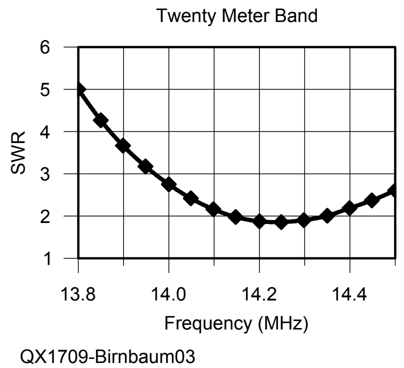

Figure 3 — SWR calculated using NEC for the 20 m band.

Finally,

$$L = \frac{X_0}{\omega_0} \text{ and } C = \frac{1}{\omega_0^2 L}.$$

The only remaining design parameter is where along the dipole arms to insert the trap. The Lopes design process gives the equations that calculate the *X* values needed for placing the trap at any location along the antenna arms subject to the constraint that the part of the antenna between the feed point and the trap must be less than a quarter wavelength at the higher frequency. In general we would like to keep as much of the center part of the full length dipole as possible since that is the part where the current, and hence the radiation, is highest. Also as we move the load towards the end of the antenna the values of the impedances needed increase rapidly.

Table 1
Design example for the 30 and 20 m bands

Antenna total length	40 feet
Antenna height	20 feet
Lower design frequency	10.1 MHz
Upper design frequency	14.05 MHz
Distance from center to trap	14 feet

Design Example

Here's a design example (see Table 1) for a 40 foot antenna that will work on the 30 and 20 meter bands. Using the above equations, the antenna has a characteristic impedance of 536 Ω. The required inductance is 2.94 µH and required capacitance is 52.7 pF.

Figures 2 and 3 shows SWR plots from a model of the above antenna using numerical electromagnetic code (NEC). To make the results more realistic, the optimal *L* and *C* values were changed to 3.0 µH and 53 pF.

Dave Birnbaum, K2LYV, holds an Amateur Extra class license. His previous calls were KN2LYV, W3GFC, WØOZX and W2IIV. Dave has a BEngr in Engineering Physics from Cornell University and MS and PhD in physics as well as an MBA from the University of Rochester. After teaching physics at the university level for several years, he transitioned to the General Railway Signal and then to Xerox Corp where he worked on digital imaging for color xerography and ink jet printing. Dave retired from Xerox as an Engineering Fellow in 2003. He holds 15 patents. In addition to ham radio, he plays the trombone in a few local concert bands. David is a member of the Tampa Amateur Radio Club and especially enjoys digital modes and CW. He has earned the WAS and DXCC awards.

Notes

[1]Luiz Lopes, CT1EOJ, "Designing a Shortened Antenna", *QST*, Oct, 2003, pp 28 – 32.

Supplement to the article “Design of a Two-band Loaded Dipole Antenna” by David Birnbaum, K2LYV (*QEX*, Sep/Oct 2017, p20-21)

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I want to thank David Brown for a series of productive and interesting discussions about the details of the calculations described in my paper. Dave and several other hams contacted me about the details of the calculation. It appears that the calculation of the impedance needed for the upper (shortening) frequency is confusing. I would like to outline a simple alternative that avoids the idea of a “negative gap.”

As David Brown has pointed out, if one uses antenna modeling programs one can iteratively adjust the value of the reactance to make the antenna appear λ/4 long. Indeed using *4NEC2* I can use the optimization function to get values that are almost the same as the calculations above. The advantage of the method I described is that one does not need to do a lot of iterative modeling to arrive at the two impedances needed to make the antenna dual band.

In Figure 1A below I sketch the antenna as it would appear if the far end were exactly λ/4 away from the center. Since the antenna is shorter than λ/4 at 10.1 MHz, there is a gap indicated by points A and B. The gap start is at the location of the trap. This gap will be replaced by an inductive reactance to make the antenna electrically λ/4 even though it is not physically that long.

At 14.05 MHz the situation is illustrated by figure 1B. Here the “outer” end of the cut is actually closer to the center than the “inner” end. This overlap, or negative gap, the amount by which the antenna needs to be shortened, is going to be replaced by a capacitive reactance which will make the antenna appear to be exactly λ/4 long.

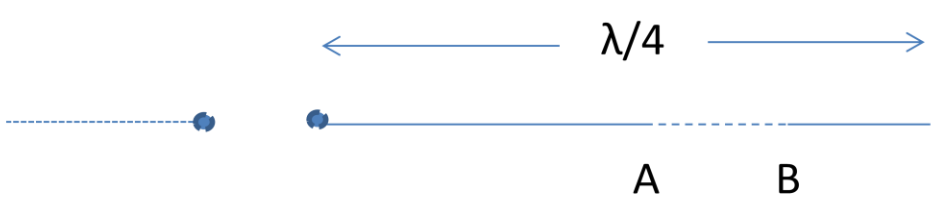

Figure 1A

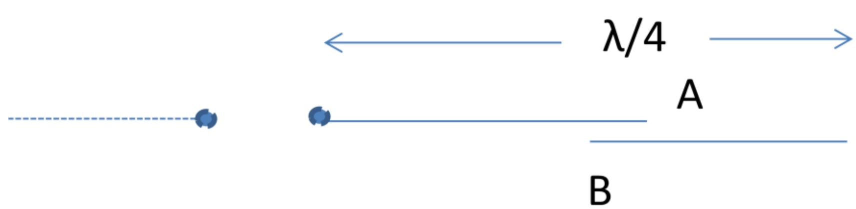

Figure 1B

I believe most of the confusion comes from measuring the impedances relative to one end of the antenna according to the Lopes' paper. The location of the outer point, point A, is known since it is just the distance from the center of the antenna to the chosen trap location. However, the computation of the location of point B relative to the center is not as clear. But a simple trigonometric substitution makes the computation much easier. If one were to measure the impedance along the transmission line model of the antenna from the end the impedance would be given by:

$$Z0 \cdot \cot(\beta')$$

where in this case β' is measured in electrical degrees *from the end*. Referring to figure 1A or 1B the impedance at the inner part (point A) is given by:

$$Z0 \cdot \tan(90 \cdot L_A/\lambda/4).$$

Where L_A is the distance from the center to the trap. The impedance at the outer, point B, is given by

$$Z0 \cdot \cot(90 \cdot L_B/\lambda/4)$$

Where L_B is the distance from the end of the antenna to the trap location.

I will use the antenna design that I gave as an example: a 40 foot dipole tuned for resonance at 10.1 and 14.05 MHz. The antenna is 20 feet above ground. The traps are placed 6 feet in from the ends of the dipole.

The value of the reactance needed is just the difference of the impedance seen at points A and B. If the antenna were exactly λ/4 long the impedance of the antenna would smoothly change from the open circuit at the end to the normal value at the center. The transmission line equation 2 in the paper [check] uses β which is the distance in electrical degrees from the end. This angle will change from 0 degrees at the end to 90 degrees at the center. In terms of the physical distance the angle beta in degrees is given by:

$$90 \cdot \text{length}/(\lambda/4)$$

Now to the specifics of the calculations.

At 10.1 MHz shown in figure 1A where point B is 6 feet from the end and point A is at 14 feet from the center where the distance from A to B, 3.17 feet, is enough to make the antenna λ/4 long. Since the λ/4 distance is 23.17 feet, these distances correspond to an electrical distance of 23 degrees to point B from the end and 54 degrees to point A from the center. A calculator gives the impedance of 1264 ohms at point B (536* cotan(23)) and 767 ohms (536*tan(54)) at point A. The difference Xb-Xa is 496 ohms which is the value of inductive reactance needed to make the antenna λ/4 electrically. If one were to simply make a shortened antenna this would correspond to an inductance of 7.8 uH.

At 14.05 MHz, λ/4 is 16.65 feet. The electrical distance from the end to point B is 32 degrees and from the center to point A is 76 degrees. Substituting these values into the transmission line equation we get the impedance at point B is 847 ohms and at point A is 2096 ohms. Note that now point A is at a higher impedance than at point B and so the difference Xb-Xa will be negative, as expected for a capacitive reactance. The difference Xb-Xa is –1249 ohms which is the value of capacitive reactance needed to electrically shorten the antenna. If one were to make a single band antenna with these dimensions this would correspond to a capacitance of 9 pF.

Once one has the impedances needed the formulas in the paper describe how to find the component values of a parallel L-C circuit that will have the desired impedances at the two frequencies of interest.

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