Hi Larry,

Readers may like to note that in the article “An Automated Method for Measuring Quartz Crystals” by Richard J. Harris, G3OTK, *QEX* Nov/Dec 2013 pp 3-8, an exact formula can in fact be written for \( C_h \). The formidable looking expressions in the text hide the fact that the algebra contains numerous cancellations. Using the valid approximation:

\[
\left( \frac{f_a}{f_0} \right)^2 - 1 \approx 2 \left( \frac{f_a}{f_0} - 1 \right)
\]

we can derive:

\[
C_s = \frac{\left( f_a - f_0 \right) C_H - \left( f_a - f_0 \right) C_C}{f_a - f_0 - f_a - f_0}
\]

\[
\approx \frac{\left( f_a - f_0 \right) C_H - \left( f_a - f_0 \right) C_C}{\left( f_a - f_0 \right) C_H - \left( f_a - f_0 \right) C_C}
\]

\[
\approx \frac{C_H - C_C}{C_H - C_C}
\]

[Eq 1]

where:

\[
C_s = \frac{C_C}{C_s + C_C}
\]

[Eq 2]

\( n = 1, 2 \) are the series capacitances as given in the text. Similar to Equation 12 in the text, which can be rewritten as Equation 3:

\[
C_m = 2 \left( \frac{f_a - f_0}{f_a - f_0} \right) (C_s + C_C)
\]

[Eq 3]

we can also easily show that:

\[
C_m = 2 \left( \frac{f_a - f_0}{f_B} \right) (C_s + C_H)
\]

[Eq 4]

Note that in the limit \( C_C \) going to infinity then \( C_s \approx C_H \) and we recover the familiar Equation 2 from the August 2007 *QST* Technical Correspondence letter by Jack Smith, K8ZOA, “Measuring Motional Parameters of a Quartz Crystal,” but it is not identical to it:

\[
C_m = 2 \left( \frac{f_a - f_0}{f_B} \right) (C_s + C_H)
\]

[Eq 5]

From the above results we can also obtain Equation 6:

\[
C_m \approx 2 \left( \frac{f_a - f_0}{f_0} \right) \left( \frac{C_H + C_C}{C_h + C_C} \right)
\]

[Eq 6]

This approximation is good for \( C_m \ll \left( C_h + C_C \right) \), which in the limit of \( C_C \) going to infinity is identical with Equation 2 from Jack Smith’s Technical Correspondence:

\[
C_m \approx 2 \left( \frac{f_a - f_0}{f_B} \right) (C_s + C_H)
\]

[Eq 7]

Using values from Figure 1 of Jack Smith’s Technical Correspondence, we find a discrepancy of just under 7% too low from the formula of Equation 6, in agreement with Jack’s remarks of a 5–15% further discrepancy in his Equation 2 from vector network analyzer measurements. Unfortunately no simple fudge factor formula as he suggested for a capacitance can be added to \( C_r \) in Equation 7 to obtain agreement with Equation 6. Using my Equation 6, we can also derive a simpler formula for \( C_r \) than my Equation 1:

\[
C_r \approx \frac{C_C^2 \left( f_2 - f_0 \right) \left[ C_{C_r} - C_H \right] - C_C \left( f_1 - f_0 \right) \left[ C_{C_r} - C_C \right]}{\left( f_1 - f_0 \right) \left[ C_{C_r} - C_H \right] - \left( f_2 - f_0 \right) \left[ C_{C_r} - C_C \right]}
\]

[Eq 8]

In view of the simpler formulas given in Equations 6 and 8, which can easily be coded into a program or spreadsheet, they can be used for finding \( C_s \) and \( C_C \) with an accuracy of at least 2%, without having to perform a numerical solution.

Note that all the formulas are subject to \( 1/Q^2 \) corrections (See the *QEX* Letters to the Editor by Alan, Bloom N1AL\(^2\) which are insignificant for high Q HF crystals but may be important for VHF and higher frequency overtone crystals. This is a subject recently investigated further by Jim Koehler, VE5FP.\(^3\)

— 73, Tuck Choy, MOTCC; m0tcc@arrl.net

Hi Larry,

I must congratulate Tuck, MOTCC, for deriving an equation for the holder capacitance of crystals. I had tried to derive such a formula myself but gave up when I realized that I could achieve the same result using a numerical method. I have used Tuck’s formula (Equation 1) with data that I measured and I can confirm that it yields the same values for holder capacitance within ± 0.1 pF. Such small differences may well be due to measurement uncertainty. In my article I mentioned that I had not read of a method of automatically measuring the motional parameters and holder capacitance of crystals. I have subsequently learned of another method that was described by Jim Koehler, VE5FP, in an article entitled “Some Thoughts on Crystal Parameter Measurement,” and which was published in the July/August 2008 edition of *QEX*, as noted by Tuck. (See Note 3.)

— 73, Richard Harris, G3OTK, 10 South St, South Petherton, Somerset TA13 5AD, United Kingdom; rj.harris.g3otk@gmail.com

Notes

1. Jack Smith, K8ZOA, “Measuring Motional Parameters of a Quartz Crystal (Technical Correspondence),” *QST* Aug 2007, pp 74-75. A straightforward derivation in a Colpitts oscillator configuration is also given by Andrew Smith, G4OEP on his website: [http://g4oep.atrace.com/crystal%20parameters/deriving_g3uur.htm](http://g4oep.atrace.com/crystal%20parameters/deriving_g3uur.htm) but the Colpitts oscillator configuration with \( C_{sec} \) assumed infinite is in fact not necessary for its derivation.
