Letters to the Editor

Experimental Determination of Ground System Performance for HF Verticals (Jan/Feb 2009 through Jan/Feb 2010)

Dear Larry,

As my series of *QEX* articles on ground system experiments were published I received a number of questions and comments. When I wrote the description of the experiment in Part 1, I only wanted to give an overview of the experiment and left out many details. Then of course there were questions about the omitted details! One frequent question concerned the power reflected from the vertical, back to the VNA due to mismatch at the antenna feed point. This mismatch varied as the ground system was changed. While most of the time this effect was very small, it still had to be taken into account, which it was for every measurement. I referred to this in Part 5 but not in detail. I also received a note from Paul Kiciak, N2PK, pointing out that I was misusing the term $|S_{21}|$ in some of the figure captions. He was right, I should have used the term transmission gain or something similar.

At the beginning of the experiments I wrote my initial thoughts in the experiment notebook. These notes were in the form of a conversation with myself, but with a little cleaning up they can be used to address the two points mentioned above. An excerpt from my notebook follows.

I hope this will help answer some of the questions.

Notebook Excerpt

The problem

In this series of experiments, I want to quantitatively determine the effect of various ground systems on the efficiency of a vertical. For example, If I add X number of radials to a given ground system, how much stronger is my signal at a distant receiver for the same power input to the antenna (P_i)? The power input to the antenna is defined as:

$$P_i = P_S - P_r \qquad \text{[Eq 1]}$$

where:
P_s = forward or incident power at the antenna feed point.
P_r = reflected power at the feed point.

I will have to keep this distinction between P_i and P_s in mind later in the data analysis!

What I'm interested in is the power gain (G_T) expressed as:

$$G_T = \frac{P_L}{P_i} \qquad \text{[Eq 2]}$$

where:
P_L = power in the receiver due to the excitation of the vertical with P_i. G_T in Equation 2 is a power ratio and I want to express it in dB:

$$G_T\,[\text{dB}] = 10\log\left(\frac{P_L}{P_i}\right)[\text{dB}] \qquad \text{[Eq 3]}$$

As I go through the experiments, I will be interested in the change in G_T [dB] from one ground system configuration to another. By taking the difference between G_T [dB] for one case and that for another I will have the "improvement" (or the lack thereof!) in dB for a given change in the ground system.

Solution to the Problem

After I had tried more conventional techniques, Paul Kiciak, N2PK, suggested a really slick way to determine G_T [dB]: use a vector network analyzer (VNA), excite the test antenna with the output of the VNA and measure the resulting signal via a remote antenna connected to the VNA input port. Repeat this for each change in the ground system.

What the VNA will give me is $|S_{21}|$ [dB] around the loop from the VNA output port to the test antenna, then out to the receiving antenna and back to the VNA detector port. $|S_{21}|$ is defined by:

$$|S_{21}| = \sqrt{\frac{P_L}{P_s}} \qquad \text{[Eq 4]}$$

where:
P_L = power in the load, in this case the input to the VNA.
P_s = power supplied by the source, in this case the output of the VNA

The VNA gives $|S_{21}|$ [dB]:

$$|S_{21}|\,[\text{dB}] = 20\log\left(\sqrt{\frac{P_L}{P_s}}\right)[\text{dB}] \qquad \text{[Eq 5]}$$

I can pull the square root out of Equation 5 so that:

$$|S_{21}|\,[\text{dB}] = 10\log\left(\frac{P_L}{P_s}\right)[\text{dB}] \qquad \text{[Eq 6]}$$

That's very nice but what I really want is G_T [dB] as given in Equation 3. I can see that Equation 3 has the same form as Equation 6, therefore:

$$G_T\,[\text{dB}] \Leftrightarrow |S_{21}|\,[\text{dB}] \qquad \text{[Eq 7]}$$

Be careful. There is a trap here! While G_T [dB] may be numerically equal to $|S_{21}|$ [dB] (Note: for the moment only, we are assuming $P_r = 0$), G_T is not the same as $|S_{21}|$! There is a conceptual difference.

A Necessary Tweak on G_T [dB]

For Equations 6 and 7 I assumed that $P_i = P_s$, $P_r = 0$. However, we know that is true only if the feed point impedance of the vertical is exactly 50 Ω (Z_0 for these experiments). In other words, $P_r = 0$, but in general that will not be true. What I want is:

$$G_T \Rightarrow |S_{21}'| = \sqrt{\frac{P_L}{P_i}} \qquad \text{[Eq 8]}$$

where:
P_i = the power delivered to the input of the test antenna and $|S_{21}'|$ is the transmission from the input of the vertical to the input of the VNA via the receiving antenna, but $|S_{21}|$ is the transmission from the output of the VNA to the input of the VNA. The problem is that what I get from the VNA is $|S_{21}|$ [dB], not $|S_{21}'|$ [dB]. Due to reflection $P_i < P_s$ and that means $|S_{21}'| > |S_{21}|$. What I need to do is to determine the difference between the two and modify what the VNA gives me, to get the G_T [dB] I'm after.

I derived an expression for this additional term, incorporating SWR measurements, and Paul sent me another expression in terms of $|S_{11}|$.

$|S_{21}|$ to $|S_{21}'|$

I know from Equation 1 that:

$$P_i = P_s - P_r \qquad \text{[Eq 9]}$$

I can eliminate P_r from Equation 9 and calculate P_i in terms of P_s and SWR at the feed point:

$$\frac{P_i}{P_s} = 1 - \left[\frac{SWR-1}{SWR+1}\right]^2 \qquad \text{[Eq 10]}$$

Using Equations 4 and 8:

$$|S_{21}'| = \sqrt{\frac{P_L}{P_i}} = \sqrt{\frac{P_L}{P_s}}\sqrt{\frac{P_s}{P_i}} = |S_{21}|\sqrt{\frac{P_s}{P_i}} \qquad \text{[Eq 11]}$$

Substituting Equation 10 into Equation 11:

$$|S_{21}'| = \frac{|S_{21}|}{\sqrt{1-\left[\frac{SWR-1}{SWR+1}\right]^2}} \qquad \text{[Eq 12]}$$

Paul sent me the following expression:

$$\frac{P_L}{P_i} = \frac{|S_{21}|^2}{1-|S_{11}|^2} \quad \text{[Eq 13]}$$

which I will change to:

$$|S_{21}'| = \sqrt{\frac{P_L}{P_i}} = \frac{|S_{21}|}{\sqrt{1-|S_{11}|^2}} \quad \text{[Eq 14]}$$

to make Equation 13 look like Equation 12.

It's not too hard to show that:

$$|S_{11}| = \frac{SWR-1}{SWR+1} \quad \text{[Eq 15]}$$

Equations 12 and 14 are equivalent. For these experiments I will use Equation 12.

One More Step

I need $|S_{21}'|$ [dB] not $|S_{21}|$. From Equation 12:

$$|S_{21}'|[\text{dB}] = |S_{21}|[\text{dB}] - 10\log\left[1-\left(\frac{SWR-1}{SWR+1}\right)^2\right] \quad \text{[Eq 16]}$$

During these experiments I will record both $|S_{21}|$ [dB] (which the VNA gives me) and SWR (which the VNA also gives me) for every ground system configuration. One small thing to watch out for:

$$\left[1-\left(\frac{SWR-1}{SWR+1}\right)^2\right] \le 1 \quad \text{[Eq 17]}$$

Therefore the log of Equation 17 will be negative. That means I will need to *add* the 10 log[**] term to the value of $|S_{21}|$ [dB] I get from the VNA. Also $|S_{21}|$ [dB] < 0 dB for these experiments. For example, if $|S_{21}|$ = –42.6 dB and the SWR is 2, –10 log[**] = +0.5 dB. Then $|S_{21}'|$ = –42.1 dB.

I will create a table of values for Equation 18:

$$K = -10\log\left[1-\left(\frac{SWR-1}{SWR+1}\right)^2\right] \quad \text{[Eq 18]}$$

The table will have all positive numbers to modify the recorded values for $|S_{21}|$ [dB] to get $|S_{21}'|$ [dB].

— *73, Rudy Severns, N6LF, PO Box 589, Cottage Grove, OR 974249;* **n6lf@arrl.net**

Hi Rudy,

Thank you for sending the detailed information about the derivation of the equations and the calculations used in your *QEX* series. There was a lot of interest in the series, and I am sure many readers will be interested to learn more about the calculations behind your results.

— *73, Larry Wolfgang, WR1B,* QEX *Editor;* **lwolfgang@arrl.org**

SDR: Simplified (Jan/Feb 2010)

Larry,

Karl-Otto Müller, DG1MFT, found a couple of mistakes I made in my last column. The drawings for Figures 3B and 3C were swapped. Also, I am not sure what happened, but when I did the *gnuPlot* work for a couple of the figures, I was off by a factor of 2 times. The 400 Hz sine wave is actually 200 Hz and the 20 kHz wave is actually 10 kHz. The principle to be illustrated is still correct: the energy in the higher Nyquist zones is due to the size of the error between the samples and the true waveform, and gets progressively worse as the sampled wave approaches one half the sampling frequency.

There is no column for the March/April issue because of problems I faced in getting the software tools to work correctly. I owe the readers a lot of code and other supporting work for previous columns. In order to make that process easier, I have created a Web site at **www.dsp-radio-resources.info** so that I can post updates from readers, answer questions from readers, and supply an alternative to the ARRL Web site for file downloads. It is meant to be a forum for information exchange for the readers.

— *73, Ray Mack, W5IFS, 17060 Conway Springs Ct, Austin, TX 78717;* **w5ifs@arrl.net**

Hi Ray,

Thanks for the information about your new Web site. I apologize for our layout error on Figure 3 of your Jan/Feb column.

— *73, Larry, WR1B*

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