John E. Post, KA5GSQ

Embry-Riddle Aeronautical University, 3700 Willow Creek Rd, Prescott, AZ, 86301; john.post@erau.edu

On Determining Loop Gain through Circuit Simulation

Loop gain is a fundamental parameter for electronic circuits that employ either positive or negative feedback. This article discusses how to determine loop gain through circuit simulation.

Amplifiers employing positive or negative feedback are fundamental building blocks in electronic circuits. Negative feedback is employed to linearize amplifiers in order to reduce distortion of the input signal and improve amplifier bandwidth. Conversely, applying sufficient positive feedback to an amplifier results in an oscillator because an output signal occurs when no input signal is present.[1]

Figure 1 depicts a block diagram for a simple amplifier with positive feedback. The gain of the amplifier is A while the gain (or loss) of the feedback network is K. The input and output voltages of the overall circuit are v_i and v_o respectively, while the voltage at the output of the feedback network is v_f. Again referring to Figure 1, the output voltage is obtained from the amplified sum of the input and feedback voltages, or

$$v_o = A\left(v_i + v_f\right) \quad \text{Eq. 1}$$

while the voltage at the output of the feedback network is found from

$$v_f = K v_o \quad \text{Eq. 2}$$

Combining Equations 1 and 2 in order to eliminate the feedback voltage term allows the voltage at the output of the amplifier to be expressed in terms of the input voltage as

$$v_o = \frac{A}{1 - A \bullet K} v_i = \frac{1}{K} \bullet \frac{1}{\frac{1}{A \bullet K} - 1} v_i \quad \text{Eq. 3}$$

In Equation 3 the term AK is referred to as the loop gain T.

In order for the positive feedback amplifier to sustain oscillations the Barkhausen criteria requires that the magnitude of T is equal to unity at a frequency where the phase of T is equal to 0 degrees or radians (or some integer multiple of 360° or 2π radians). In order for oscillations to *start* the loop gain must be greater than unity at a frequency where the phase is 0 degrees or radians (or some integer multiple of 360° or 2π radians).[2] Thus, the loop gain provides critical information concerning whether an electronic circuit will actually function as an oscillator.

[1]Notes appear on page 7.

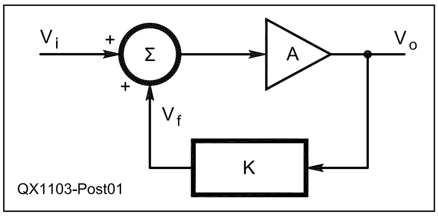

Figure 1 — Block diagram of positive feedback amplifier depicting input voltage v_i, output voltage v_o, and feedback voltage v_f.

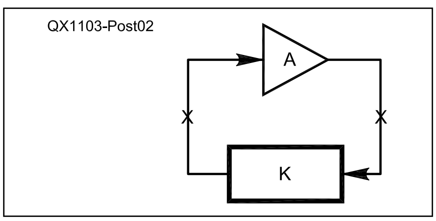

Figure 2 — Positive feedback amplifier redrawn to emphasize the feedback loop. Each "X" indicates locations where it is possible to break open the loop in order to determine loop gain.

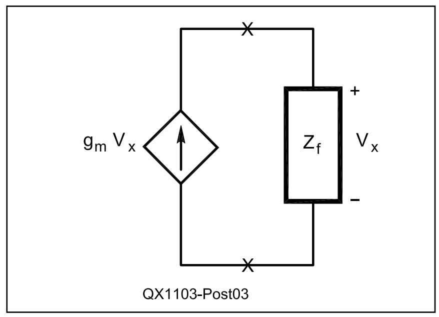

Figure 3 — Alternative model of a positive feedback amplifier consisting of a dependent current source with current $g_m v_x$ in parallel with a feedback impedance Z_f. Each "X" indicates locations where it is possible to break open the loop in order to determine loop gain.

In order to develop a method to determine the loop gain, consider Figure 2, which is redrawn to emphasize the feedback portion of the loop from Figure 1. If the loop is opened at either "X" and a test voltage v_{test} is applied to the side of the loop that leads to the amplifier input, what voltage $v_{measure}$ is measured at the other side of the loop opening? Obviously $v_{measure} = AKv_{test}$ so that the loop voltage gain is $v_{measure}/v_{test} = AK = T$.

Figure 3 shows a circuit model of the positive feedback amplifier, Figure 2. As shown in the figure the circuit consists of a dependent source, in this case a voltage controlled current source that represents an ideal transconductance amplifier and an impedance Z_f that represents the load of the feedback network on the controlled source. In general it is possible to define either a loop voltage gain T_v or a loop current gain T_i. The procedure for determining either one is the same—the loop is opened, a test voltage (current) is applied and a response voltage (current) is determined and then their ratio is obtained. But now a second load impedance Z_f is connected across the source in order to maintain the same load impedance on the source with the loop open as with the loop closed. This is shown in the two circuits shown in Figure 4. For the circuit on the left side of the figure applying a test voltage of v_x results in a voltage of v_y and the loop voltage gain is $T_v = v_y/v_x = g_m v_x Z_f/v_x = g_m Z_f$. For the circuit on the right side of the figure applying a test current of i_x results in a current of i_y and the loop current gain is $T_i = i_y/i_x = g_m(v_x/Z_f) = g_m Z_f$. In this simple example $T_v = T_i = g_m Z_f = T$ which is the overall loop gain, but this result holds only if the output impedance of the dependent source is ignored as we shall see.

Although the technique of opening the loop appears simple in theory there are a number of factors that complicate its application to a practical circuit. For example opening the loop may remove the dc bias current for the active device so that it no longer functions. Another problem is precisely duplicating the value of the load impedance that appears in parallel with the dependent source once the loop is broken.

An Experimental Method for Determining the Loop Gain

In 1975 R.D. Middlebrook proposed a method to overcome these limitations for negative feedback amplifiers.[3] Middlebrook applied this method to measure the loop gain of negative feedback amplifiers. It is also possible to apply the method to determine the loop gain of negative feedback amplifiers through circuit simulation.[4]

In general Middlebrook's method determines the loop voltage gain T_v by inserting an arbitrary voltage source v_z into the loop as shown by the circuit on the right side of Figure 5. Because practical dependent sources contain an internal impedance Z_s this impedance, as well as the load impedance Z_f due to the feedback network, is shown in the circuits in Figure 5.

Because the internal impedance of the voltage source v_z is a short circuit the load impedance seen by the dependent source is unchanged and bias currents are unaffected. All that remains is to obtain the voltages v_x and v_y, through either measurement or circuit simulation, and then determine the loop voltage gain $T_v = v_y/v_x$.

The loop current gain T_i may be obtained by the dual of the previous method as shown in the left side of Figure 5. This time an arbitrary current source i_z is added in parallel with the dependent source and feedback network. Because the internal impedance of the current source is infinite the load impedance seen by the dependent source is unchanged and the bias currents are unaffected. In this case it is necessary to obtain the currents i_x and i_y, through either measurement or circuit simulation, and then determine the loop current gain $T_i = i_y/i_x$.

Next it is necessary to determine the overall loop gain T from the loop voltage gain T_v and loop current gain T_i. This is demonstrated in Appendix 1, which modifies Middlebrook's result for the case of a positive feedback amplifier (oscillator). As shown in Appendix 1, once the loop voltage gain T_v and the loop current gain T_i are obtained the overall loop gain T can be found from

$$T = \frac{T_v T_i - 1}{T_v + T_i - 2} \qquad \text{Eq. A-1}$$

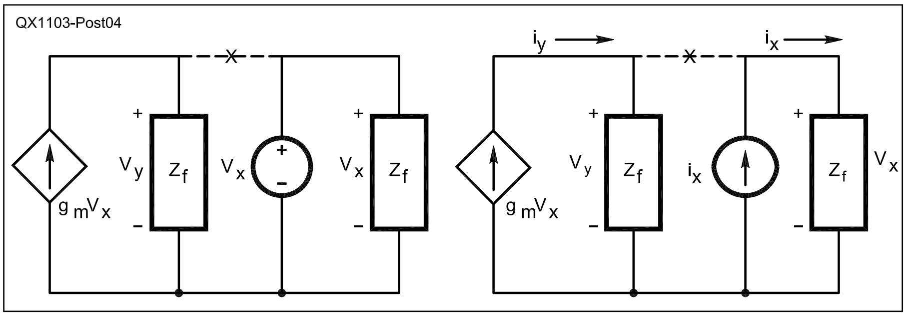

Figure 4 — In order to obtain loop voltage gain break loop at "X", insert a test voltage source, and then determine $T_v = v_y/v_x$ (left circuit). In order to obtain loop current gain break loop at "X", insert a test current source, and then determine $T_i = i_y/i_x$ (right circuit). In each case after the loop is broken it is necessary to insert a replacement feedback impedance Z_f in order to maintain a constant load for the dependent current source.

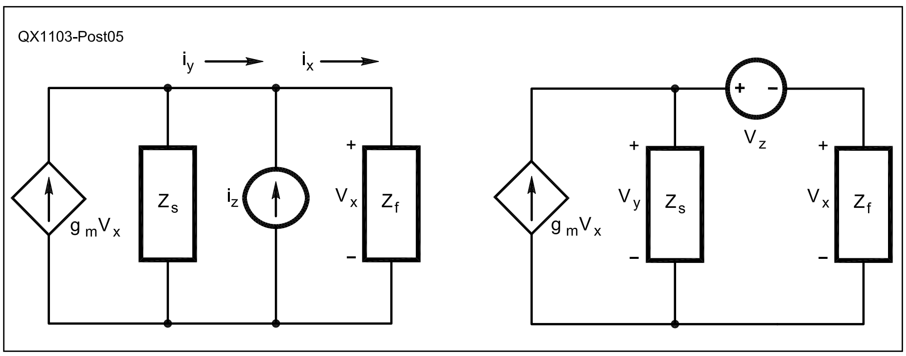

Figure 5 — Determine loop voltage gain by inserting an arbitrary voltage source v_z and then obtaining $T_v = v_y/v_x$ (right circuit). Determining loop current gain by inserting an arbitrary current source i_z and then obtaining $T_i = i_y/i_x$ (left circuit).

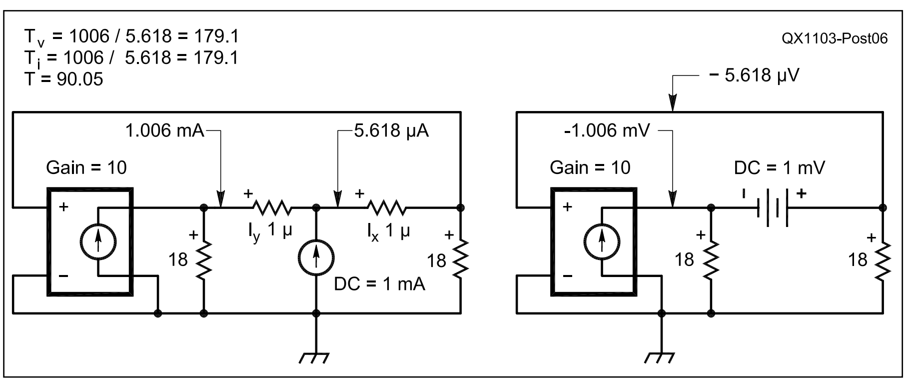

Figure 6 — Circuit simulation to determine loop voltage and current gain for the circuits shown in Figure 5 for the case of $Z_s = R_s = 18\ \Omega$ and $Z_f = R_f = 18\ \Omega$.

For large values of T_v and T_i Equation A-1 can be approximated as

$$T \cong \frac{T_v T_i}{T_v + T_i}$$

which demonstrates that, like the formula for two parallel resistors, the overall loop gain T will be smaller than the smallest of either the loop current gain T_i or the loop voltage gain T_v.

The balance of this article will demonstrate how to apply Equation A-1 through circuit simulation in the case of dc, audio, and radio frequency positive feedback amplifiers.

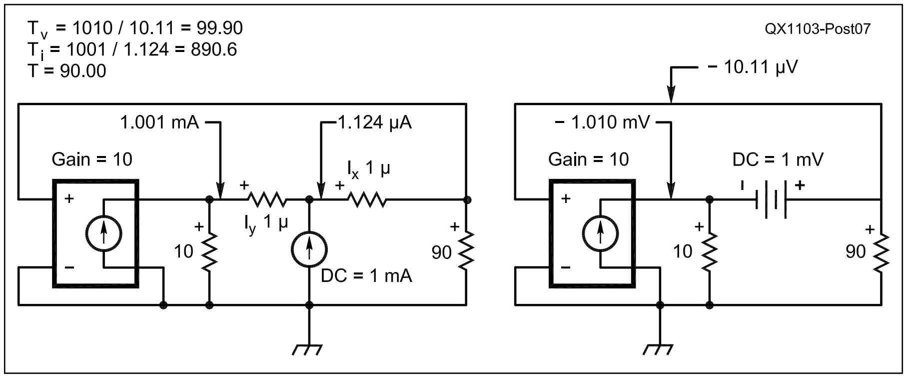

Figure 7 — Circuit simulation to determine loop voltage and current gain for the circuits shown in Figure 5 for the case of $Z_s = R_s = 10\ \Omega$ and $Z_f = R_f = 90\ \Omega$.

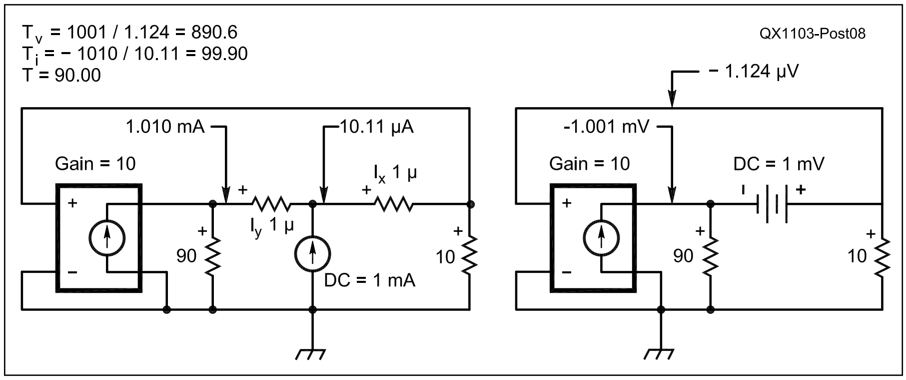

Figure 8 — Circuit simulation to determine loop voltage and current gain for the circuits shown in Figure 5 for the case of $Z_s = R_s = 90\ \Omega$ and $Z_f = R_f = 10\ \Omega$.

Example: Determining the Loop Gain for DC Circuits

Because Equation A-1 was derived by applying general circuit laws, it applies equally to either time varying (ac) or constant (dc) voltages. Thus before applying the method to determine the loop gain of an oscillator it is instructive to review several dc simulations to demonstrate that Equation A-1 is correct, as well as illustrate the method of obtaining the loop gain T through simulation.

Figure 6 presents a circuit simulation of a positive feedback amplifier. In this case $R_s = R_f = 18\ \Omega$ so the load and feedback resistances are equal and the theoretical loop gain is $T = g_m R_{eq} = 10S(18\Omega \| 18\Omega) = 90$ V/V. The function of the two 1 μΩ resistors is to sense the currents i_x and i_y without perturbing the results. As shown in the figure in the case of equal load and feedback resistance $T_v = T_i = 179.1$. Applying Equation A-1 with these values yields $T = 90.05$ V/V, so excellent agreement is obtained between the simulated and theoretical values of the overall loop gain T. Notice that the value of either the test voltage or current source in Figure 6 is immaterial since only the ratio of i_y and i_x or v_y and v_x is of interest rather than the individual values of voltage or current.

Figure 7 presents a circuit simulation for the case of $R_s = 10\ \Omega$ and $R_f = 90\ \Omega$ so the theoretical loop gain is unchanged because $T = g_m R_{eq} = 10S(10\Omega \| 90\Omega) = 90$ V/V. In this case the source resistance is much less than the feedback resistance so the dependent source approximates a voltage source because of its relatively low resistance and the loop voltage gain dominates the overall loop gain T because $T_i = 890.6$ while $T_v = 99.90$. Applying these values to Equation A-1 again results in T = 90.00 V/V, so excellent agreement is once again obtained between the simulated and theoretical values of the loop gain and since $T_v << T_i$, $T \cong T_v$.

Figure 8 presents a circuit simulation for the converse of the previous case in that now $R_s = 90\ \Omega$ and $R_f = 10\ \Omega$. In this case the feedback resistance is much less than the source resistance so the dependent source approximates a current source. The theoretical loop gain is unchanged because $T = g_m R_{eq} = 10S(90\Omega \| 10\Omega) = 90$ V/V. The results obtained here are the reverse of the previous results and the loop current gain dominates because $T_v = 890.6$ while $T_i = 99.90$. Applying these values to Equation A-1 once again results in T = 90.00 V/V, so excellent agreement is obtained between the simulated and theoretical values of the loop gain and since $T_i << T_v$, $T \cong T_i$.

Example: Determining the Loop Gain of a Wein-Bridge Oscillator

The results in the previous section are instructive because they point out that if the feedback amplifier can be approximated as either an ideal voltage or current amplifier the overall loop gain T may be obtained from a single circuit simulation. In order to illustrate this concept consider Figure 9 which presents the schematic diagram of an audio frequency Wien-bridge oscillator. Referring to the figure, the oscillator consists of an operational amplifier with two feedback paths. The upper negative feedback path through the 1kΩ and 5kΩ resistors has a gain of 1 + 5k/1k = 6 V/V. This path provides negative shunt feedback to the amplifier output

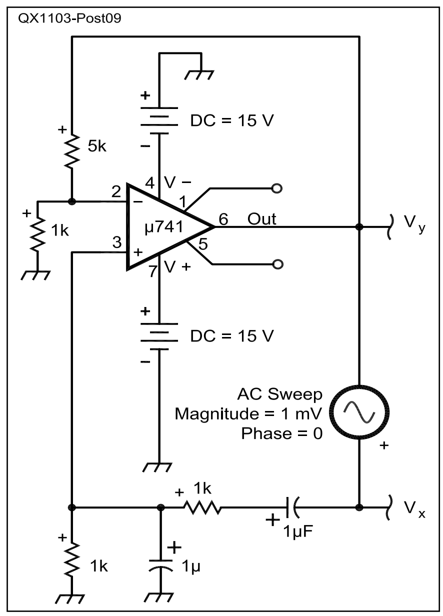

Figure 9 — Circuit simulation to determine the loop voltage gain for the Wein-bridge oscillator. The upper network provides negative feedback and controls the amplifier gain. The lower network provides positive feedback and controls the frequency of oscillation. In this case the loop is broken and an arbitrary voltage source is inserted between the output of the amplifier and the input of the feedback network.

Table 1
Calculated versus simulated results for Colpitts oscillator loop gain and oscillation frequency. The error is obtained from (Calculated-Simulated)/Calculated x 100%.

	Loop Gain (V/V)			*Oscillation Frequency (MHz)*		
C (pF)	*Calculated*	*Simulated*	*% Error*	*Calculated*	*Simulated*	*% Error*
340	1.00	1.06	-6.00%	1.0184	1.0157	0.27%
373	1.07	1.13	-5.61%	1.0160	1.0135	0.25%
426	1.17	1.22	-4.27%	1.0124	1.0102	0.22%
470	1.24	1.28	-3.23%	1.0097	1.0076	0.21%
556	1.34	1.38	-2.99%	1.0049	1.003	0.19%
800	1.50	1.52	-1.33%	0.9929	0.9913	0.16%

which reduces the amplifier output resistance by the factor of $1 + AK$ so the circuit very closely approximates an ideal voltage amplifier.[1]

The lower positive feedback path through the *RC* network controls the frequency of oscillation. In the special case where the two resistors are of equal value and the two capacitors are of equal value it is possible to show that the oscillation frequency is

$$f_o = \frac{1}{2\pi RC}$$

while the gain of the lower feedback network is 1/3 V/V[5]. Thus for the circuit in Figure 9, the oscillation frequency is

$$f_o = \frac{1}{2\pi \cdot 10^3 \cdot 10^{-6}} = 159.15 \text{ Hz}$$

and the loop gain is

$$6 \cdot \frac{1}{3} = 2 \text{ V/V.}$$

Figure 10 shows the phase in degrees (top) and magnitude in V/V (bottom) of $T_v = v_y/v_x$, where v_y and v_x were obtained by conducting an ac sweep between the frequencies of 155 Hz and 165 Hz of the circuit shown in Figure 9. The cursor in the top plot in Figure 10 is positioned to show that the phase is 0º at f_o = 158.92 Hz while the bottom plot shows the loop voltage gain T_v = 1.999 V/V at that same frequency. A separate simulation (not shown) reveals that the loop current gain is T_i = 44.175 kA/A at the same frequency so the circuit is an almost perfect voltage amplifier as claimed previously. Thus $T_v << T_i$, so $T \cong T_v$ = 1.999 V/V. In this case the theoretical and simulated loop gain and resonant frequency differ from each other by no more than a few tenths of a percent in either case. These results further confirm the utility of this method to determine oscillator loop gain through simulation.

Example: Determining the Loop Gain of a Colpitts Oscillator

As an example of a more general and sophisticated application of this method

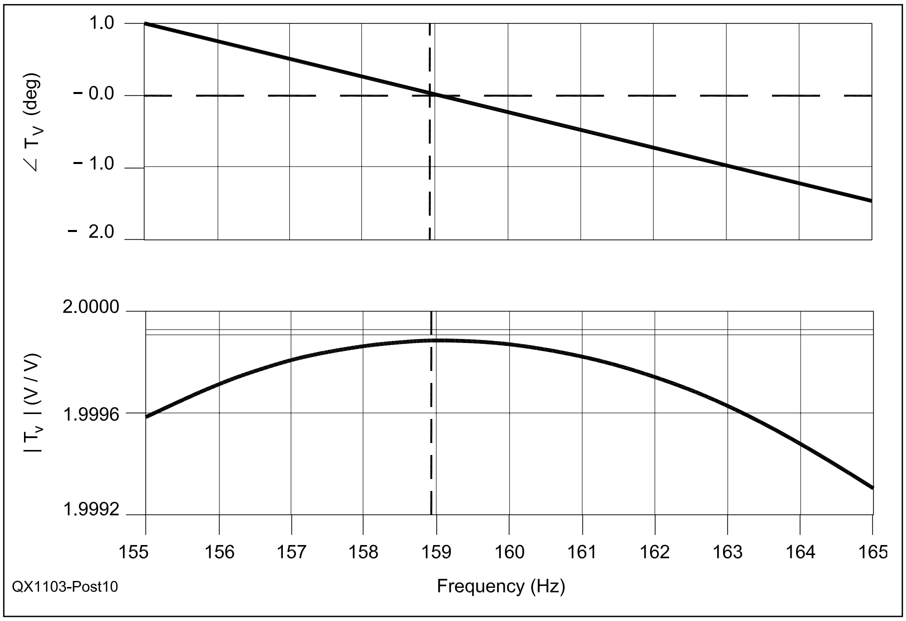

Figure 10 — Loop gain phase (top) and magnitude (bottom). The loop voltage gain T_v = 1.9999 V/V and T_y = 0° at 158.92 Hz.

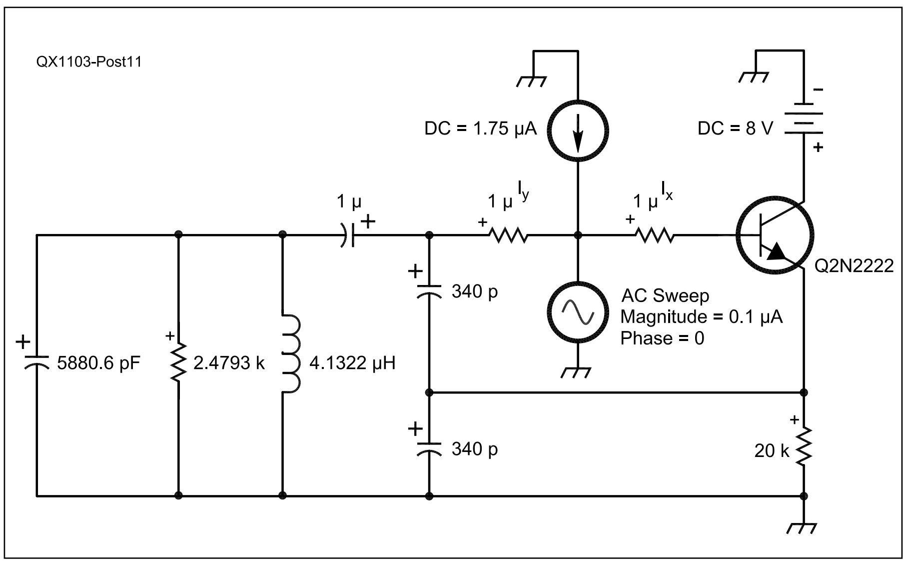

Figure 11 — Circuit simulation to obtain the loop current gain T_i of the Colpitts oscillator. In this case an arbitrary current source is connected between the input of the amplifier and the output of the feedback network.

consider the SA/NE 602 IC Colpitts oscillator circuit that appeared in the March/April 2010 issue. In Appendix 1 of that article I argued that the impedance looking into the base of the transistor was so high that it could be approximated as an open circuit. For this reason it was possible to open the loop at the base of the transistor and ignore the loading effect of the base of the transistor on the feedback network, which simplifies the analysis of the oscillator circuit. The method presented here of determining loop gain through circuit simulation provides a means for verifying that assumption and validating the results presented in that article.

Figures 11 and 12 depict the schematic diagram of the Colpitts oscillator shown in Figure 2 of the previous article using the component values for the MF band oscillator shown in Table 1 of the previous article. The 2N2222 transistor is not the device used internally in the SA/NE 602 IC, but it is satisfactory for the purpose of conducting a circuit simulation to determine the loop gain in the MF band. The 1.75 µA current source was chosen to set the emitter current to 0.25 mA in order to provide the proper bias for the transistor. For the circuit in Figure 11 an ac sweep was conducted to obtain the loop current gain $T_i = i_y/i_x$ while for the circuit in Figure 12 an ac sweep was conducted to obtain the loop voltage gain $T_v = v_y/v_x$. Then the overall loop gain T was obtained through application of Equation A-1 and the phase and magnitude of T plotted as shown in the top and bottom plots respectively in Figure 13.

It is possible to examine the plots in Figure 13 in order to determine the resonant frequency f_o and the loop gain T at that frequency in the same way as was done for the loop gain results that were obtained from the Wein-bridge oscillator. These results are shown in Table 1 which compares the values of loop gain and oscillation frequency obtained through circuit simulation for the six values of the voltage boosting capacitor C with those presented in Table 3 of the previous article. As shown in the table the error between the calculated and simulated results for the loop gain are in error by no more than a few percent while the error between the calculated and simulated results for the oscillation frequency are in error by no more than a few tenths of a percent.

These results validate the assumptions detailed in the March/April 2010 article and confirm the accuracy of the formulas that were derived in Appendix 1 of that article.

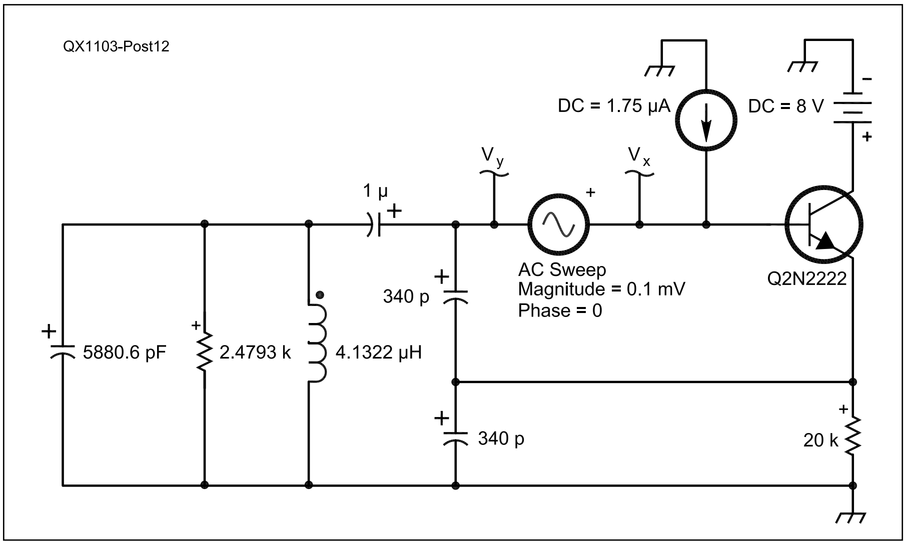

Figure 12 — Circuit simulation to obtain the loop voltage gain T_v of the Colpitts oscillator. In this case the loop is broken between the input of the amplifier and the output of the feedback network and an arbitrary voltage source is inserted in the loop.

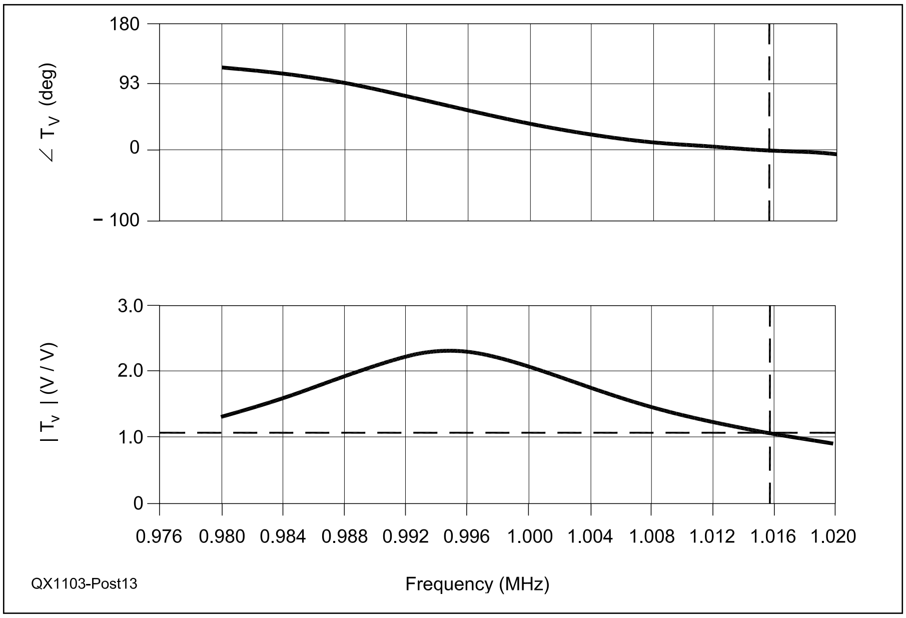

Figure 13 — Plot of Colpitts oscillator loop gain found by applying Equation A-1 to obtain the phase (top) and magnitude (bottom) of the loop gain T.

Conclusion

This article demonstrates that it is possible to apply circuit simulation techniques in order to determine the loop gain for feedback amplifiers that apply positive feedback. These results are useful to either investigate a feedback amplifier's performance or to confirm analytical results. This method was applied to determine the loop gain through simulation of dc, audio, and radio frequency circuits. In each case the error between theoretical predictions and the results obtained from simulations are no worse than a few percent.

John E. Post is an assistant professor of electrical and computer engineering with Embry-Riddle Aeronautical University in Prescott, AZ. He holds an Amateur Extra class license, KA5GSQ, and has BS, MS, and PhD degrees in electrical engineering.

Notes

[1]Comer, David J., *Modern Electronic Circuit Design,* Addison-Wesley, Reading, Massachusetts, 1976.

[2]Young, Paul H., *Electronic Communication Techniques*, 5th Edition, Pearson Prentice Hall, Upper Saddle River, New Jersey, 2004.

[3]Middlebrook, R.D., "Measurement of Loop Gain in Feedback Systems", *International Journal of Electronics*, vol. 38, no. 4, pp. 485–512, 1975.

[4]Tuinenga, Paul W., *SPICE: A Guide to Circuit Simulation and Analysis Using PSpice,* Prentice-Hall, Inc. Englewood Cliffs, New Jersey, 1988.

[5]Alexander, Charles K. and Sadiku, Matthew N.O., *Fundamentals of Electric Circuits,* 4th Edition, McGraw Hill, Boston, Massachusetts, 2009.

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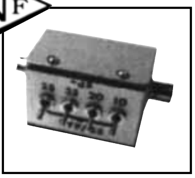

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Appendix 1

Referring to the circuit on the right side of Figure 5, summing currents entering the bottom node gives

$$-g_{\mathrm{m}} v_{\mathrm{x}} + \frac{v_{\mathrm{y}}}{Z_{\mathrm{s}}} + \frac{v_{\mathrm{x}}}{Z_{\mathrm{f}}} = 0. \qquad \textbf{Eq. A-2}$$

Then, rearranging Equation A-2 and solving for the loop voltage gain results in

$$T_{\mathrm{v}} = \frac{v_{\mathrm{y}}}{v_{\mathrm{x}}} = g_{\mathrm{m}} Z_{\mathrm{s}} - \frac{Z_{\mathrm{s}}}{Z_{\mathrm{f}}}. \qquad \textbf{Eq. A-3}$$

The overall loop voltage gain is $T = g_{\mathrm{m}}(Z_{\mathrm{s}}||Z_{\mathrm{f}})$. Modifying Equation A-3 to introduce this factor yields

$$T_{\mathrm{v}} = \frac{v_{\mathrm{y}}}{v_{\mathrm{x}}} = \frac{g_{\mathrm{m}} Z_{\mathrm{s}} Z_{\mathrm{f}}(Z_{\mathrm{s}}+Z_{\mathrm{f}})}{Z_{\mathrm{f}}(Z_{\mathrm{s}}+Z_{\mathrm{f}})} - \frac{Z_{\mathrm{s}}}{Z_{\mathrm{f}}} = \mathrm{T}\left(1 + \frac{Z_{\mathrm{s}}}{Z_{\mathrm{f}}}\right) - \frac{Z_{\mathrm{s}}}{Z_{\mathrm{f}}}. \qquad \textbf{Eq. A-4}$$

Solving for the impedance ratio Z_s/Z_f gives

$$\frac{Z_{\mathrm{s}}}{Z_{\mathrm{f}}} = \frac{T_{\mathrm{v}} - T}{T-1}. \qquad \textbf{Eq. A-5}$$

Now referring to the circuit on the left side of Figure 5, by equating currents it is possible to express i_y as

$$i_{\mathrm{y}} = g_{\mathrm{m}} v_{\mathrm{x}} - \frac{v_{\mathrm{x}}}{Z_{\mathrm{s}}}. \qquad \textbf{Eq. A-6}$$

Then, applying Equation A-6 in order to solve for the loop current gain results in

$$T_{\mathrm{i}} = \frac{i_{\mathrm{y}}}{i_{\mathrm{x}}} = \frac{g_{\mathrm{m}} v_{\mathrm{x}} - \frac{v_{\mathrm{x}}}{Z_{\mathrm{s}}}}{\frac{v_{\mathrm{x}}}{Z_{\mathrm{f}}}} = g_{\mathrm{m}} Z_{\mathrm{f}} - \frac{Z_{\mathrm{f}}}{Z_{\mathrm{s}}}. \qquad \textbf{Eq. A-7}$$

Again, the overall loop voltage gain is $T = g_{\mathrm{m}}(Z_{\mathrm{s}}||Z_{\mathrm{f}})$. Modifying Equation A-7 to introduce this factor yields

$$T_{\mathrm{i}} = \frac{i_{\mathrm{y}}}{i_{\mathrm{x}}} = \frac{g_{\mathrm{m}} Z_{\mathrm{f}} Z_{\mathrm{s}}(Z_{\mathrm{f}}+Z_{\mathrm{s}})}{Z_{\mathrm{s}}(Z_{\mathrm{f}}+Z_{\mathrm{s}})} - \frac{Z_{\mathrm{f}}}{Z_{\mathrm{s}}} = T\left(1 + \frac{Z_{\mathrm{f}}}{Z_{\mathrm{s}}}\right) - \frac{Z_{\mathrm{f}}}{Z_{\mathrm{s}}}. \qquad \textbf{Eq. A-8}$$

This time solving for the impedance ratio Z_f/Z_s gives

$$\frac{Z_{\mathrm{f}}}{Z_{\mathrm{s}}} = \frac{T_{\mathrm{i}} - T}{T-1}. \qquad \textbf{Eq. A-9}$$

Equating Equation A-5 and the reciprocal of Equation A-9 in order to eliminate the impedance ratio Z_s/Z_f produces

$$(T-1)^2 = (T_{\mathrm{i}} - T)(T_{\mathrm{v}} - T). \qquad \textbf{Eq. A-10}$$

Finally, solving Equation A-10 for the overall loop gain T gives the desired result.

$$T = \frac{T_{\mathrm{v}} T_{\mathrm{i}} - 1}{T_{\mathrm{v}} + T_{\mathrm{i}} - 2}. \qquad \textbf{Eq. A-11}$$

QEX