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An Optimum Height for an Elevated HF Antenna

*What is the best height for your antenna?
The author considers factors that can help you decide.*

There are two ways to think about antenna and propagation problems in linear media: in transmit mode and in receive mode. By the reciprocity theorem both methods will predict the same performance. We will view the problem of finding an optimum height for HF antennas in receive mode rather than in transmit mode, because this reveals very interesting insights. For example, the field-strength at the receiving location is the result of an interference pattern between waves that arrive by a direct path added to the wave reflected from the earth's surface. The addition of these two waves results in a standing wave versus height for the field strength at the receiving location. Because this vertical standing wave has peaks and can have deep nulls, there is an optimum placement for an antenna. In the equivalent transmit mode point of view, far-field transmit patterns are calculated as an interference pattern between the direct wave and a ground reflected wave, but as *The ARRL Antenna Book* explains, that point of view obscures the physical meaning of "take-off" angle, so we can't directly appreciate what happens when an antenna is elevated.[1] By viewing the problem in receive mode, however, we see, among other things, that waves arriving from the lowest arrival angle do not always result in the best link margin to a DX station. We can also see that low antennas can work surprisingly well for DX, and that the best height for vertically polarized antennas is not the same as for horizontally polarized antennas.

With this analysis it is easy show that the optimum antenna height depends on frequency, polarization, properties of the earth at the reflection point, and on the arrival angle from the wave source in the ionosphere. While surface roughness is considered, there is also a terrain dependence, which for simplicity will not be considered here; see Dean Straw's terrain analysis program HFTA in the 21st edition of *The ARRL Antenna Book*. Furthermore, since the apparent wave earth reflection point is usually distant from the antenna, it is not important what the earth looks like directly under an elevated antenna. What is important is the earth's properties at the reflection point — typically hundreds to thousands of meters distant from the tower. This is an idealized problem where we allow for surface roughness, but we assume an earth that is smooth enough so that we can apply spherical earth geometry.

We begin by laying a foundation based on a spherical earth geometry for the propagation of waves to the receiving location. The reflection properties of ground and sea water are shown to affect how the reflected wave combines in constructive and destructive interference with the direct wave. Optimum heights are found for desired ranges of arrival angles and for multiple bands. Finally, path link margins are estimated for multi-hop propagation. We discover that a range of "take-off" angles must be accommodated for optimum performance.

Spherical Earth Geometry

Because we are dealing with distances that approach the earth's horizon, we calculate the direct and earth-reflected paths using spherical-earth reflection geometry. The solution to the spherical earth geometry given in Chapter 2 of M. I. Skolnik's *Radar Handbook* involves a cubic equation to find the arc distance G_b to the reflection point.[2]

$$2G_b^3 - 3GG_b^2 + \left[G^2 - 2a_e\left(h_{ant} + h_i\right)\right]G_b + 2a_e h_{ant} G = 0 \quad \text{[Eq 1]}$$

where:

h_{ant} is the height at the receiving antenna,

a_e is the earth's radius,

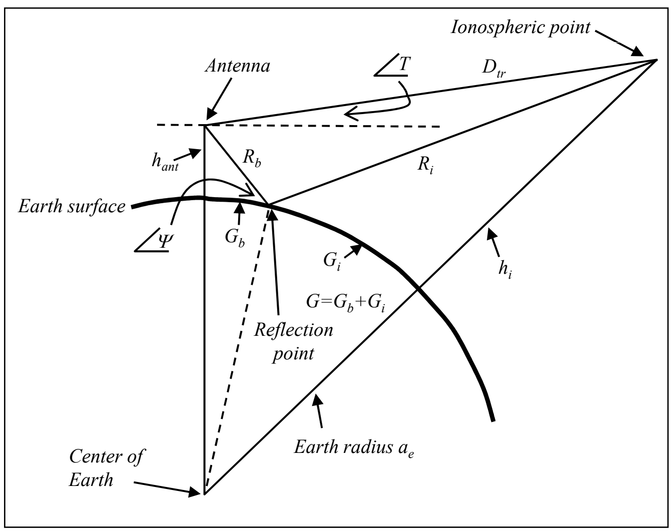

Figure 1 — Spherical earth geometry, shown with an exaggerated height dimension. Source: based on information from *Radar Handbook* (see Note 2).

[1]Notes appear on page 38.

and the distances G and G_b are functions of the angle T between the local horizon and the direction to the wave source point at height h_i in the ionosphere. Figure 1 shows the spherical earth reflection geometry and identifies all of the parameters.

The angle T is also called the "take off angle" and the "local elevation angle." See the ARRL website files update to *The ARRL Antenna Book*.[3] The direct wave arrives along path D_{ir}, and the reflected path includes distance R_i from the ionosphere to the earth reflection point and R_b from the reflection point to the receiving location. The reflection occurs at the arc distance G_b from the base of the antenna tower, and as the direct wave arrival angle T deceases, then the arc distance to the reflection point increases. Our chief concern is with the difference in the path lengths,

$$\Delta R = (R_b + R_i - D_{ir}) \quad \text{[Eq 2]}$$

and with the surface reflection coefficient at the reflection point because these determine the nature of the field variation versus height, h_{ant}.

Reflection Coefficients and Combined Waves

The plane wave reflection coefficients Γ_H for horizontal and Γ_V for vertical polarization are used to find the reflection from land or sea on a spherical earth. (See Chapter 6 of *Radiowave Propagation and Antennas for Personal Communications*.[4]) The reflection coefficient is modified by the divergence factor D and surface roughness S_r factor. The wave divergence factor is:

$$D = \left[1 + \frac{2G_bG_i}{a_eG\sin\psi}\right]^{-1/2} \quad \text{[Eq 3]}$$

where ψ is the angle of incidence on the earth's surface. The surface roughness factor is:

$$S_r = \exp(-r)\mathrm{I}_0(r); \quad r = 2\left(kh_{sd}\sin(\psi)\right)^2 \quad \text{[Eq 4]}$$

where:
I_0 is the modified Bessel function
$k = 2\pi f / c$ is the wave number
f is the signal frequency in Hz
c is the speed of light in m/s.

The roughness factor for the reflected wave is based on a roughness factor originally derived for a ratio of rough-sea to smooth-sea reflection, and is applied here generally to an earth reflection. The surface roughness parameter h_{sd} is the standard deviation of the surface height distribution in the reflection region. The complete reflection coefficients are thus $\Gamma_H S_r D$ and $\Gamma_V S_r D$ for a rough spherical earth. The reflected term fields are also multiplied by $d = D_{ir} / (R_b + R_i)$ to account for the difference in free space loss due to the differential distance between the direct and reflected waves.

For this study we will assume that horizontally polarized power is added to vertically polarized power in a ratio, P_{HV}. For substantially horizontally polarized waves, P_{HV} is chosen here to be between 10 and 20, and for substantially vertically polarized waves, P_{HV} is between 0.005 and 0.01. The polarization impurity primarily results in a slight reduction of the depths of nulls in the vertical standing wave patterns. The two polarization components are added as power because the polarization is decomposed by the ionosphere into elliptical polarization, (see *Ionospheric Radio Propagation*[5]) and reflections from a rough surface are generally random and time-variable. The expression for the signal power, P normalized to the free space value, of the combined waves at the receiving height, h_{ant} is:

$$P = \frac{P_{HV}\left[1+\exp(-jk\Delta R)\Gamma_H S_r Dd\right]^2 + \left[1+\exp(-jk\Delta R)\Gamma_V S_r Dd\right]^2}{1+P_{HV}} \quad \text{[Eq 5]}$$

The unity terms in each of the brackets represent the direct wave amplitude, and the remaining terms are the reflected wave, each in ratio to the free space value. The phase difference, $k\Delta R$, along with the phase of the reflection coefficients conspire to produce the vertical standing wave pattern of the field strength at the receiving location. *This is before any antenna is placed at the receiving location*. Since the earth's radius is large compared with the height of the ionosphere, angles T and ψ are nearly the same value, despite the exaggerated view in Figure 1. Since antenna free space elevation patterns for a level antenna are essentially symmetrical in elevation about the local horizontal plane, the direct wave entering the antenna from angle T above the horizontal plane is weighted by the same antenna pattern gain value as the reflected wave entering the antenna from angle ψ below the horizontal plane. Note also that the earth's horizon is *slightly below* the elevated antenna horizontal plane.

Expected Angles of Arrival

We will be optimizing our solution over a desired range of arrival angles. Expected arrival angles T for waves from the ionosphere for HF Propagation are available in *The ARRL Antenna Book* product notes files on the ARRL website for HF (see Note 3). For example, the combined 80 m to 10 m arrival angle statistics between Florida (FL) or Massachusetts (MA) and all regions of the World are shown in Figure 2.

Those statistics show that half the arrival angles are less than 6°, and that 90% of the arrival angles are smaller than 16°. So for HF cases, we will confine our interest to arrival angles between 2 to 16°. Viewed in transmit mode, this is the *range* of "take-off" angles that must be accommodated. Similar curves may be derived for 6 m band sporadic-*E* propagation. Notably, in the July and August 2009 "World Above 50 MHz" *QST* column, Gene Zimmerman, W3ZZ, comments on the work of Joe Kraft, CT1HZE, suggesting that arrival angle probabilities for 6 m band sporadic-*E* are bimodal, with one peak at ~5° and another at ~10° with very little below 3° or 4° or above ~13° or 14°.[6,7] Thus, arrival angles of 3° to 14° emerge as a range of interest for 6 m sporadic-E operations. Also see my article, "Optimum Height for an Elevated Communications Antenna," in *DUBUS* magazine.[8] While different from HF in the specifics, the angle ranges of interest are similar, and justify the range between 2° and 16°.

Location of the Reflection Point

The distance G_b to the reflection point on the earth's surface is solved by Equation 1 as a function of receiving point height. There is only a very

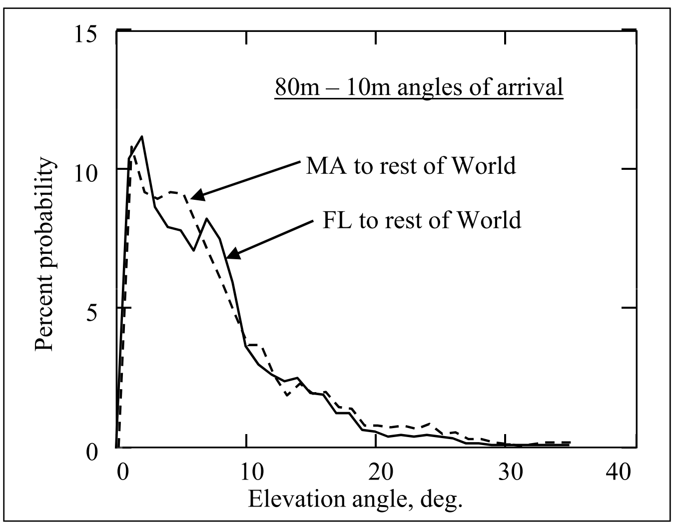

Figure 2 — Composite probability of arrival angles.

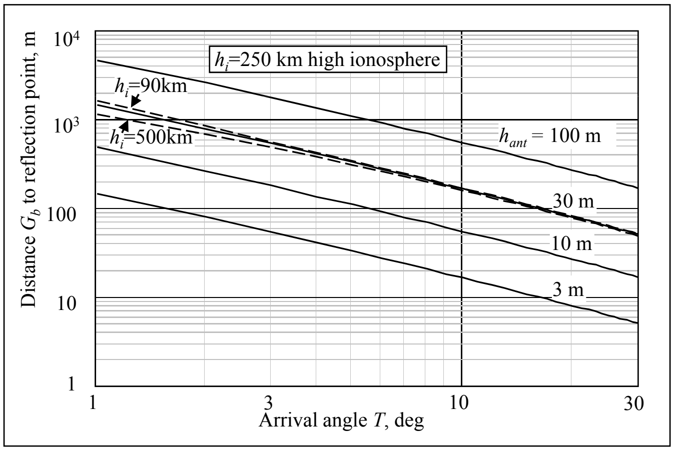

Figure 3 — Distance to the reflection point is tens to thousands of meters.

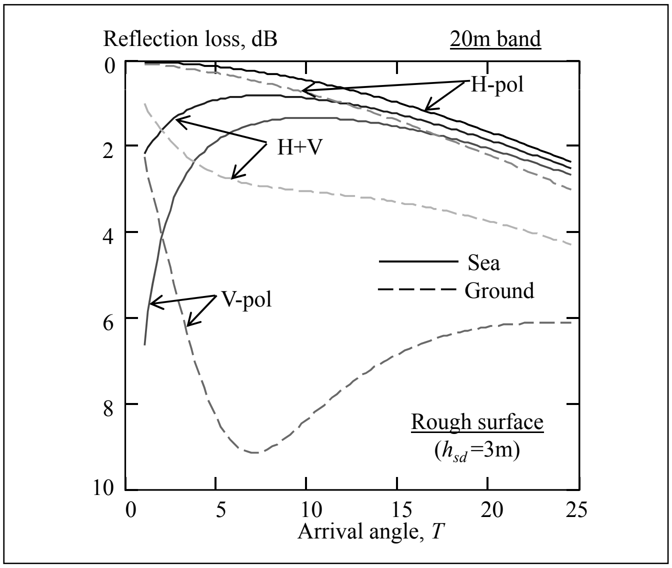

Figure 4 — Reflection coefficient with surface roughness, 20 m band.

weak dependency on the height of the ionosphere; heights from 90 km to as much as 500 km, the range of heights for the E, E_S, and F layers of the ionosphere, give very nearly the same geometrical result. There is, however, a strong dependency on the receiving height location. Figure 3 shows the distance to the reflection point versus the arrival angle for several receiving heights between 3 and 100 m with a 250 km high ionosphere. The 30 m high antenna distances are also shown (dashed lines) for 90 km and 500 km high ionosphere. *Since the reflection point is typically from a few kilometers to tens of meters away the ground immediately below the antenna does not affect elevated antenna performance*. A very good approximation to the reflection point distance is:

$$G_b = \frac{55 h_{ant}}{T} \qquad \text{[Eq 6]}$$

where:
h_{ant} is the antenna height in meters
T is the arrival angle in degrees.

The reflection point given by Equation 6 is the same as for the transmit case; please see "The Effect of Ground in the Far Field" in Chapter 3 of *The ARRL Antenna Book* (see Note 1). It should be noted that transmit patterns computed in the presence of the ground often quoting a "take off angle," *implicitly assume that, the ground is flat to beyond the distance given by Equation 6*. Here, in contrast, recall that we have allowed for a ground roughness factor.

Earth Reflection Loss

The ground or sea reflection loss, L_{earth} in dB for *multiple hop paths* can be found by setting the direct wave "1" terms to zero in Equation 5 and expressing the result in decibels. Figure 4 shows the loss in the 20 m band for horizontal, vertical and a 50% mix of the polarization, for reflection from the sea and from a medium earth (ε = 12) versus the angle T. The reflection includes a surface roughness factor of 3 m. For $2 \le T \le 16°$ this reflection loss can amount to more than 1 dB for horizontal polarization, but as much as 9 dB for vertical polarization reflected from earth ground.

Optimum Antenna Height

We can now solve Equation 5 at various frequencies, polarizations, ground constants and as a function of the height of an antenna. The specific antenna pattern — that is, the free space pattern — is not important as long as the elevation plane beamwidth is sufficient

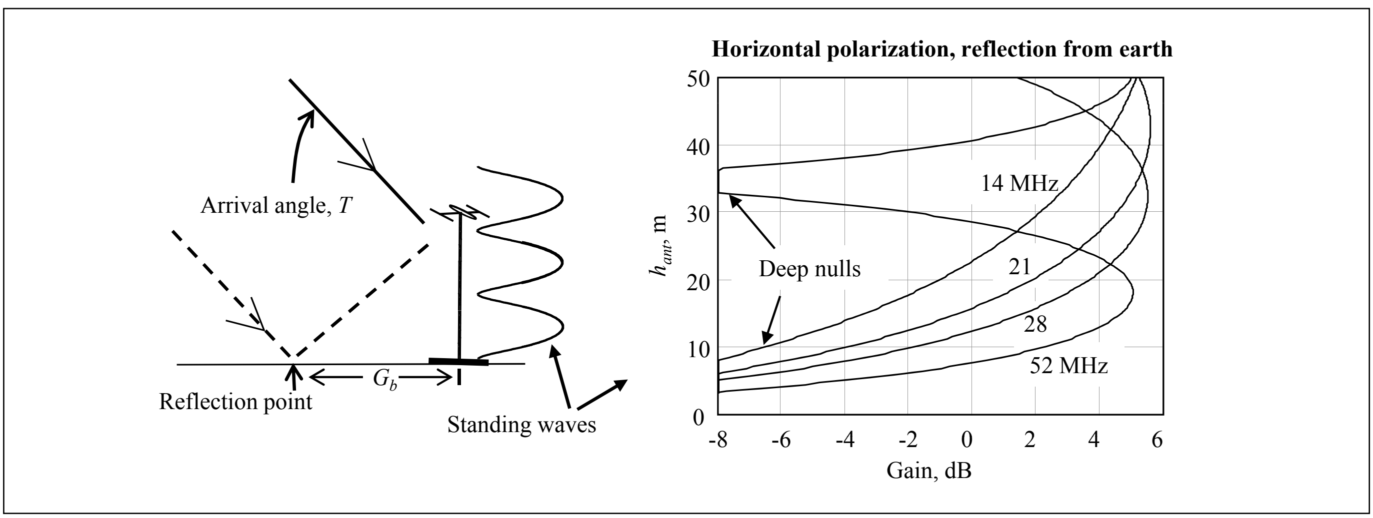

Figure 5 — Horizontal polarization (P_{HV} = 20), earth ground, T = 5°, roughness is 3 m.

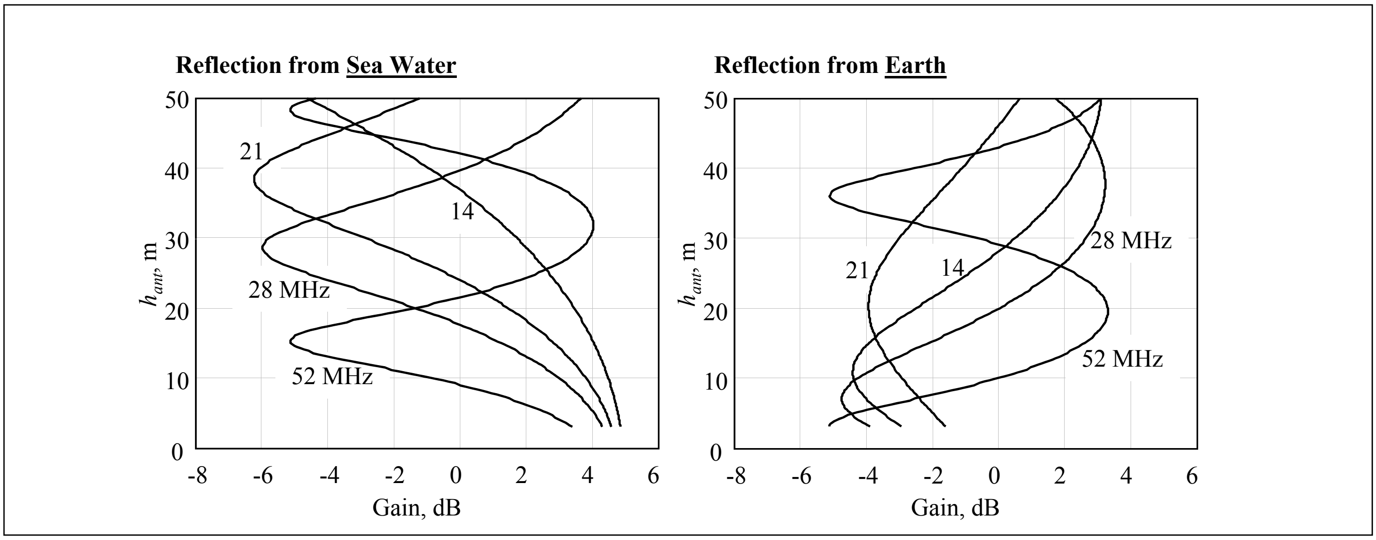

Figure 6 — Vertical polarization (P_{HV} = 0.05), T = 5°, roughness is 3 m, reflections from (left) sea water and from (right) earth ground.

to include the important angles of arrival, both above and below the local horizontal plane. We do note, however, that as the angle T increases, the waves arrive in pairs above and below the main beam peak, so that the full antenna gain for directive antennas cannot be always be realized — especially for very high gain (narrow elevation plane beamwidth) antennas.

Figure 5 shows the geometry and the calculated vertical standing wave patterns produced by the interaction of the direct and earth reflected waves for earth ground parameters $\varepsilon = 5$ and $\sigma = 0.005$ S/m. The standing wave peaks and nulls depend on frequency and on arrival angle, here 5°. This suggests placing the antenna at the signal peak, which is one definition of the optimum antenna height.

Results for horizontally polarized waves reflected from the sea differ primarily in the depth of nulls compared with earth ground reflected results of Figure 5. There are transmitter mode equivalents to the receive mode standing wave patterns shown in Figure 5. The transmit mode patterns are computed in the presence of a ground, and usually a peak "take-off angle" is identified; see for example Figure 3 in the companion article in the June 2011 issue of *QST*.[9] Clearly the transmit mode patterns do not make it easy to identify the best height for the antenna.

Figure 6 shows the vertical polarization performance for reflection from sea water $\varepsilon = 70.6$ and $\sigma = 4.54$ S/m, on the left and from ground with $\varepsilon = 5$ and $\sigma = 0.005$ S/m on the right. The saline water model is from *Radiowave Propagation and Antennas for Personal Communications* (see Note 4). The sea-reflected, vertically polarized case has an optimum at sea level. This is why vertically polarized antennas on the beach are so effective on some DXpeditions such as during the VP6DX operation. Note that the optimum heights per frequency for vertically polarized antennas with the reflection from earth ground are not the same as for horizontal polarization. Ground mounted vertical antennas with a reflection from earth ground will have negative height gains of –1 to –5 dB. The gains shown in Figures 5, 6 and 7 are in addition to any free space directive gain provided by the antenna system. Results in Figures 5 and 6 are exactly analogous to the results that have been predicted and measured to within a decibel at open air test sites in the 30 to 932 MHz range. See Section 6.3 in *Radiowave Propagation and Antennas for Personal Communications* (see Note 4).

Concentrating now on the 20 m band, Figure 7 shows field-strength signal levels relative to the free space value for reflections from the ground. *These are not antenna patterns but rather signal field strength levels that are then sampled by an antenna*. The axes have been flipped compared with the previous figures. The upper dashed asymptote is the *maximum constructive interference* for the continuum of all arrival angles between 2 and 16°. Specific results for 2°, 5°, 10° and 15° are shown by the embedded curves. The lower dashed asymptote is defined by the *destructive interference* for the continuum of arrival angles. The lower asymptote intersects the 2° arrival angle curve at a cusp, which defines an optimum antenna height for that frequency. At that elevation, the height gain, g_w has the smallest variation versus the range of arrival angles, and its minimum gain value is the highest. When an antenna is placed there, the actual free space antenna gain, at the pattern elevation angle, T, adds to this field strength height gain. Antennas that are higher than the optimum height will encounter degraded performance at the higher angles of arrival because the nulls defining the lower asymptote to the right of the cusp are likely to be a factor. This is why in some cases a lower antenna can significantly outperform a higher antenna. If we had chosen a higher minimum required arrival angle, the optimum height would decrease. Similar curves can be drawn for other HF bands or combinations of bands, and optimum heights can be found.

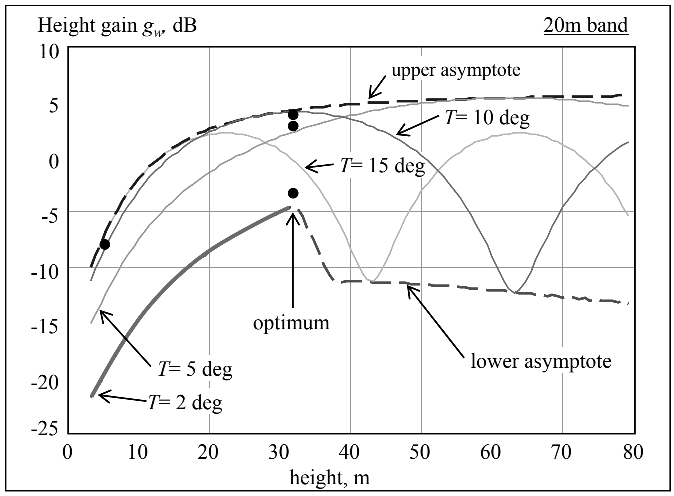

Figure 7 — Height gain for horizontal polarization in the 20 m band.

Multiband Considerations

Since the geometry of the reflection point, including divergence and surface roughness, are fixed in physical dimensions, the vertical interference patterns don't quite scale with wavelength. Thus, the optimum height does not scale exactly with frequency. Some multiband Yagi beams can cover the 40 m to 6 m bands in a single structure. Raising and lowering such an antenna is not usually desirable, so knowing an overall optimum height could be very useful. A family of curves like the 20 m band curves in Figure 7 can be calculated for any frequency band or any combination of frequency bands.

One effective strategy for finding an overall optimum over multiple bands is to choose the best height for the highest frequency band of interest. That somewhat sacrifices the performance for the lowest arrival angles at the lower frequency bands, but more gently than the destructive interference loss of height gain for higher arrival angles if a higher antenna were chosen.

The optimum heights for various frequency bands between 7 and 54 MHz are shown in Figure 8. The three curves are for three different minimum angles, the upper curve shows optima for a 1° to 16° arrival angle range, the middle curve for 2° to 16°, and the lower curve for 3° to 16°. The middle curve slopes from about 1.5 to 1.6 wavelengths between 7 and 29 MHz.

If operation anywhere in the 10 m to 40 m bands is of equal interest, the "best" height works out to about 19.9 m. That height is suitable for arrival angles as low as 1° in the 10 m band, and is also suitable for angles above about 4° in the 20 m band. In the 40 m and 30 m bands the results are "best effort," but as will be shown in the next section, paths at higher arrival angles may exist, but with an increased number of earth-ionosphere hops. If the 20 m band is to be optimized, then the best height is about 32 m. If 6 m band operation is important then the optimum height is about 15.3 m. The heights between about 15 m and 32 m (50 to 105 ft) emerge as a good range of compromise choices for multiband HF and 6 m band operations.

This analysis also provides some insight into the physical basis for the operation of phased Yagi antennas mounted at different heights on a tower. By combining the signals from the two or more Yagis using phase shifters, it is possible to enhance gain in the direct-wave path while minimizing the destructive interference from the earth reflection. Possibly significant performance improvement might be realized.

Path Link Considerations

Many details are important in calculating a path link at HF, but for illustration here we examine a simplified path where both ends of the link are located on relatively flat (but not smooth) terrain, and the ionosphere and earth are suitable for the needed reflections along the path. Path link margin depends on the height of the ionosphere, h_i as well as on the arrival angle, T. Figure 9 shows the hop distances for several ionospheric heights as a function of the arrival angle over a spherical earth. For our example we will assume that the ionospheric refraction and reflection occurs at an effective height of 250 km. So a 10,000 km path might be traversed with 3, 4 or 5 hops, each 3,333 km or 2,500 km or 2,000 km respectively. Other paths are possible as well, as Davies described in *Ionospheric Radio Propagation* (see Note 5). The three different hops are marked by the shaded circle in Figure 9, with corresponding marks in Figure 7. Different hop distances mean different arrival angles, which affects the total path loss.

The wave interference gain, or height gain, g_w in dB shown in Figure 7 applies to each end of the link. Ionospheric reflection/refraction loss is L_{ion} in dB and can be as little as 2 to 5 dB.[10] In this simplified example, we will use 3 dB to account for polarization decomposition, as described by Davies (see Note 5). The free space loss is $27.6 + 20 \log(2\, D_{tr} \times f)$ dB for one hop, where the frequency,

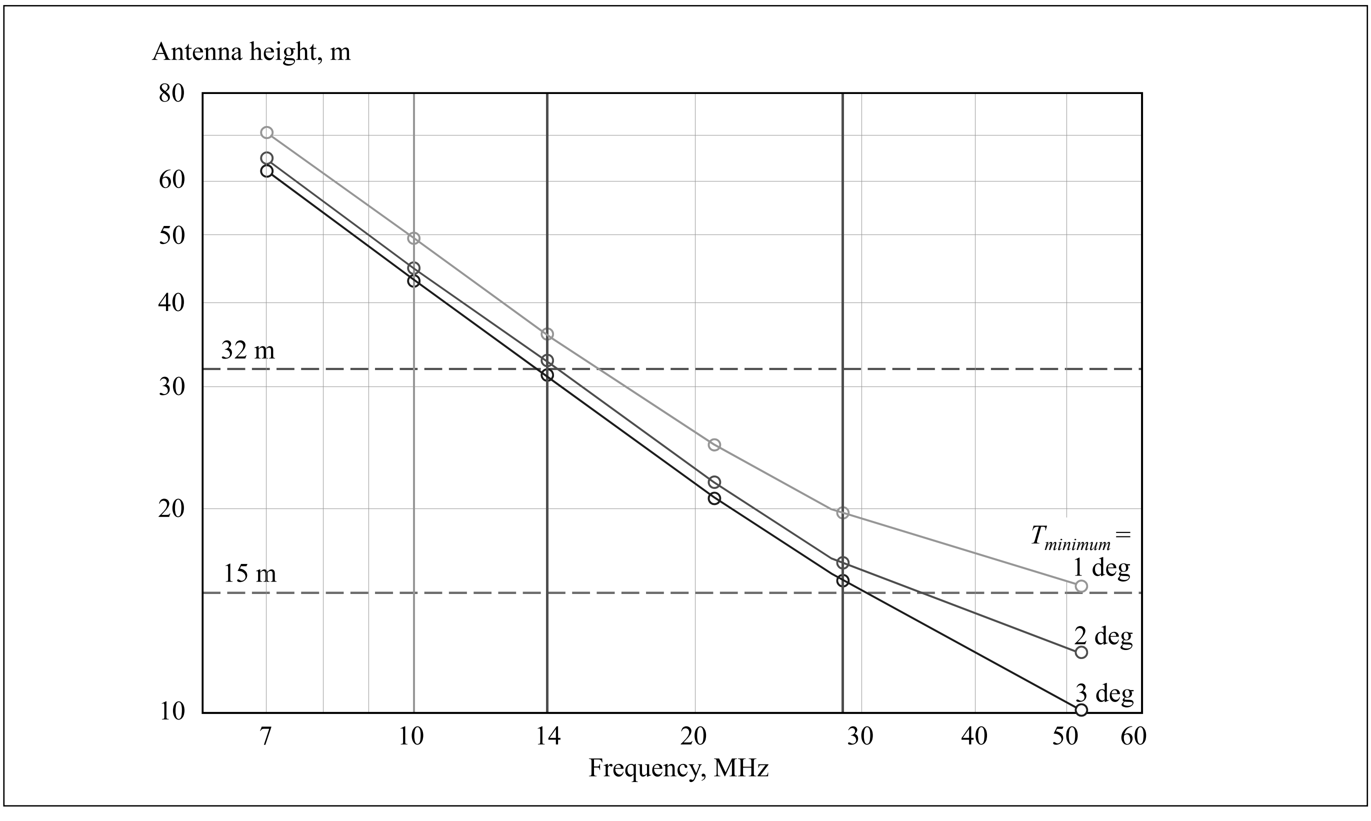

Figure 8 — Optimum antenna heights over even terrain for various frequencies.

Table 1
Path Losses in a 10,000 km Path for Different Numbers of Hops.

Hops	*T (deg)*	*First hop loss (dB)*	*Height gain (dB)*	*Rest of hops loss (dB)*	*PL (dB)*	*S-units*
3	2.8	[126.1 + 3]	–[–4 – 4]	{ 9.6 + 7.1}	153.8	3.6
4	6.9	[123.7 + 3]	–[+3 + 3]	{10.6 + 8.1 + 7.1}	146.4	4.8
5	10.4	[121.8 + 3]	–[+4 + 4]	{11.7 + 9.2 + 8.2 + 7.6}	157.8	2.9

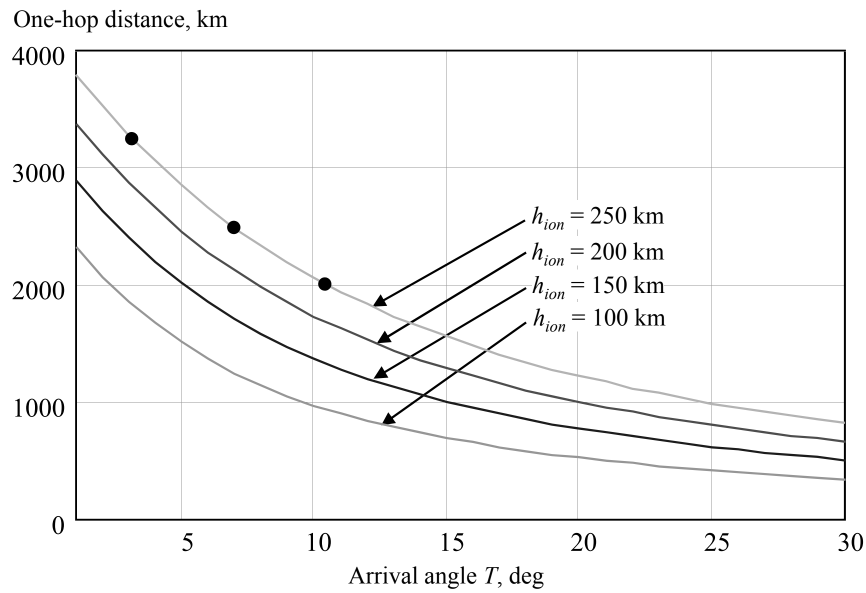

Figure 9 — Hop distances, with the 3, 4, and 5 hop points marked for a 10,000 km path.

f, is in MHz and the distance, D_{tr} is in meters. Each additional j^{th} hop adds an incremental free space loss, an earth reflection loss, $L_{earth,j}$ (from Figure 4), and another ionospheric reflection loss, $L_{ion,j}$. The path loss for *n* hops is written in Equation 7 so that the bracketed terms are for a single hop or first hop, including wave interference at the link ends *A* and *B*. The braces contain additional losses for hops 2 through *n* if present.

$$L_{path} = \left[27.6 + 20\log(2D_{tr}f_{MHz}) + L_{ion} - g_{w,A} - g_{w,B}\right] + \cdots$$
$$+\left\{\sum_{j=2}^{n}\left(20\log\left(\frac{j}{j-1}\right) + L_{ion,\ j} + L_{earth,\ j}\right)\right\}$$

[Eq 7]

Our example path in the 20 m band with a 250 km effective ionospheric height might require 3 to 5 or more hops to traverse a 10,000 km path. The various gains and losses for this idealized example are listed in Table 1. In general, several of these as well as other possible paths will exist, causing fading and signal variations as the ionosphere changes. Table 1 shows the path losses and estimated received S-units for 50 W transmitted power (approximately 100 W PEP for CW or processed SSB) and with 32 m high dipoles at each end. Gain antennas will improve signals in proportion to the antenna gains. The bracketed and braced terms in Table 1 correspond to the same terms in Equation 7.

Notice that the four-hop path has a stronger signal by over an S-unit more than the example three-hop path because the increased height gains g_w of a combined 8 dB at the higher arrival angle (the difference between the top and bottom solid circles at the optimum height in Figure 7) at both ends of the link more than compensate for the additional reflection losses of an additional hop. The height gain is the intersection of the arrival angle, *T*, with the antenna height in Figure 7. The four-hop 6.9° arrival angle results in less destructive interference by 7 dB at each end of the link than the three-hop 2.8° arrival angle. *The lowest arrival angle path is not always the best!* Agonizing over a lower "take-off angle" is futile. This effect justifies a compromise lower limit for the angle of arrival at lower frequencies when choosing a compromise height for a multiband antenna. The five-hop path suffers additional earth and ionospheric reflection losses, but still results in a respectable S = 2.9 signal.

Suppose that the antenna at one end of the link is lowered to 5m. The height gain, g_w becomes –17 dB for the 2.8° three-hop path, so that path is not viable. The gain is –8 dB for the four-hop path, however, which is 12 dB lower than at the optimum height, resulting in an S = 1 reading. That is still a –115 dBm signal, which is suitable for CW as well as SSB. This result helps to explain the occasional spectacular DX results possible from low and indoor attic antennas.[11] If the arrival angle is, say >5°, the low antenna captures signals that are not dramatically worse than from a high antenna. Indeed, KE4PT has earned WAS-TPA and DXCC, now with 200 confirmed entities as well as a 6 meter VUCC from southern Florida, using just an indoor antenna.

Uncertainties in the ionospheric reflection/refraction loss increase as the number of hops increases, and Equation 7 represents a best case value. Link reliability can be estimated by attaching variances to the several propagation loss components and by using the method of Hagn described in Section 8.4 of *Radiowave Propagation and Antennas for Personal Communications* (see Note 4).

Summary and Conclusions

Constructive and destructive wave interference from a direct path and an earth reflected path causes a vertical standing wave at the antenna location. The standing wave pattern details depend on the wave angle of arrival, polarization, on whether the reflection point was ground or sea water, and on the terrain profile (not considered here). Optimum antenna heights are largely governed by the lowest arrival angle deemed important at the highest desired frequency. Antennas that are placed too high can suffer from significant wave destructive interference at desired higher arrival angles. The earth reflection point is typically several kilometers away for low arrival angles, but can be tens of meters for very high arrival angles, so the condition of the ground immediately below an elevated antenna is of little importance. Because height gain can be significantly greater for higher arrival angles, the lowest arrival angle path (fewest hops) does not always result in the best link margin for paths that can be closed with different numbers of earth-ionosphere hops. Optimum height is 1.5 to 1.6 wavelengths for any one band, or a compromise height can be found for a multiband antenna operating over several bands by using the optimum for the highest frequency. Keeping in mind that this analysis was limited to rough, but not locally mountainous earth, nor a dense urban region, antenna heights in the range of 15 m to 32 m (50 to 105 ft) are found to be reasonable compromise choices for multiband antennas operating from a fixed height.

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Notes

[1]R. Dean Straw, N6BV, Ed., *The ARRL Antenna Book*, 21st Edition, 2007.

[2]M. I. Skolnik, *Radar Handbook*, Second Edition, McGraw-Hill Professional, 1990.

[3]See the *Antenna Book* tab at **www.arrl.org/product-notes**, updated statistical elevation-angle files, accessed 29 December 2010.

[4]K. Siwiak, Y. Bahreini, *Radiowave Propagation and Antennas for Personal Communications*, Third Edition, Artech House, Norwood MA, 2007.

[5]K. Davies, *Ionospheric Radio Propagation*, National Bureau of Standards Monograph 80, Washington DC, 1965.

[6]Eugene Zimmerman, W3ZZ, "Long Distance E_s Propagation on 50 MHz at Solar Cycle Minimum — Part 1," The World Above 50 MHz, *QST*, July 2009, p 88.

[7]Eugene Zimmerman, W3ZZ, "Long Distance E_s Propagation on 50 MHz at Solar Cycle Minimum — Part 2," The World Above 50 MHz, *QST*, August 2009, p 86.

[8]K. Siwiak, "Optimum Height for an Elevated Communications Antenna," *DUBUS Magazine*, Vol. 39, 3rd Quarter 2010, pp 86-99.

[9]K. Siwiak, "What's the Optimum Height for an HF Antenna?," *QST* June 2011.

[10]Recommendation ITU-R P.534-4.

[11]K. Siwiak, KE4PT, "You Can Enjoy DXing with a Modest Station," *CQ Amateur Radio*, November 2010, pp 46-48.

QEX