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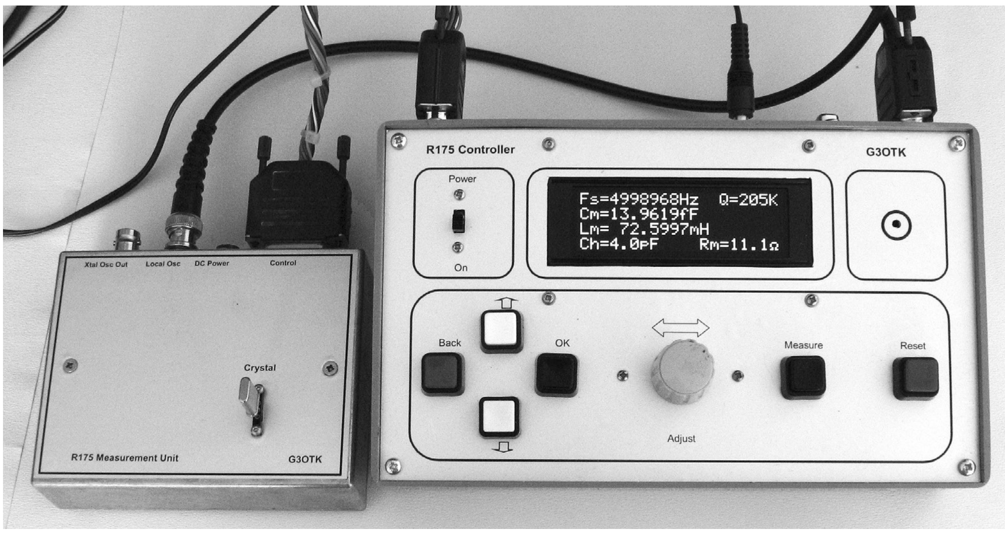
An Automated Method for Measuring Quartz Crystals

G3OTK proposes a method for automatically measuring all of the components of the equivalent circuit of quartz crystals using a Colpitts oscillator.



Ladder crystal filters have become popular in home-brew transceivers and QRP kits because of the availability of very low cost crystals. It seems to me that many projects use filters that have not been designed, but have been constructed on the "if it sounds right then it is right" principle. Although software for the design of ladder filters is available, a filter cannot be designed if the crystals have not been characterized and the equivalent circuit components, or motional parameters, determined with some accuracy.

Many methods for measuring the equivalent circuit of quartz crystals have been proposed over the years. These can be divided into two classes: those requiring a stable signal generator as an excitation source and those where the crystal is used in an oscillator. This latter class has the advantage that the principle item of test equipment is a frequency counter, which few serious experimenters will be without. Many years ago G3UUR proposed a simple method using a Colpitts oscillator, and many references can be found to it on the Internet.[1] As originally proposed, this method only gives a ball-park figure for the motional capacitance because it does not take into account the two capacitors of the Colpitts oscillator, nor the holder capacitance of the crystal. Also, it does not give any information about the motional resistance of the crystal or its series resonant frequency.

All of the methods for evaluating the motional parameters of quartz crystals that I have read about in Amateur Radio magazines require the measurements to be entered into formulae by hand to give the final parameter values. Making the measurements and undertaking the subsequent calculations for, say, 100 crystals is time consuming.

This article describes a further development of the oscillator technique that not only gives an accurate figure for the motional capacitance, motional inductance, holder capacitance and series resonant frequency but also gives a good estimate for the motional resistance. A microprocessor controls the measurements, makes the calculations, shows the results on an organic light emitting diode (OLED) display and also sends them to a computer running a spreadsheet program. Each crystal can be characterized in a few seconds and the results sorted into groups those with similar properties. The tabulated data will also give an insight into the spread of the motional parameters associated with inexpensive crystals. These are intended to be used in oscillators, and the information required for filter design is not part of the specification.

[1]Notes appear on page 8.

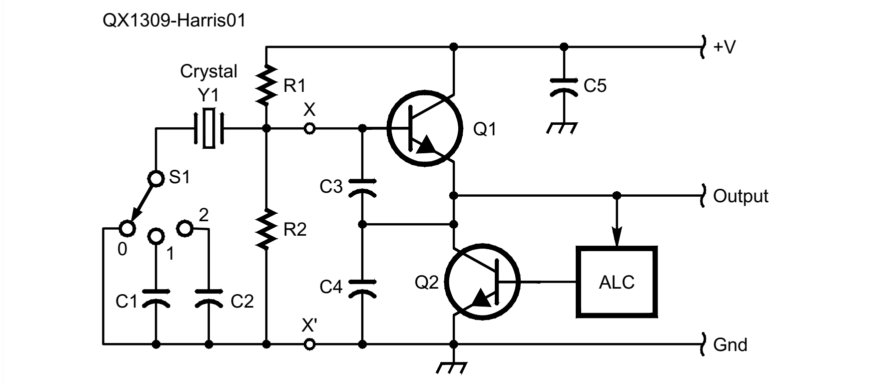


Figure 1 — This schematic is a basic crystal test circuit.

Outline of the Measurement Method

The outline is shown in Figure 1. The crystal under test, X1, is connected in series with a switch, SW1, which selects 0 V, C1 or C2. Q1, C3 and C4 form a Colpitts oscillator with the amplitude maintained at 1 mV by an Automatic Level Control (ALC) loop. At this low level, Q1 operates in a linear and predictable manner. For the amplitude to be constant, the motional resistance of the crystal must be balanced precisely by the negative resistance generated by the Colpitts Oscillator. This is proportional to the emitter current and gives a means of determining the motional resistance.

The loading effect of the bias resistors R1 and R2 is very small and can be ignored. At a frequency, f , the input impedance of the Colpitts oscillator at points X-X', as seen by the crystal, are given by Equation 1.

$$Z_{in} = B\left(\frac{-q\ I_e}{(2\pi f)^2\ C3\ C4\ K\ T}\right) - \frac{j}{2\pi f}\left(\frac{1}{C3}+\frac{1}{C4}\right) \quad \text{[Eq 1]}$$

where:
K is Boltzmann's constant (1.3807 × 10^{-23} m^2 kg s^{-2} K^{-1})
T is the temperature in kelvins
q is the charge on an electron (1.6022 × 10^{-19} C)
I_e is the emitter current.

I have added the constant B to make a first order correction for the effects of the reduced current gain of Q1 at frequencies typical of the crystals used in CW and SSB filters. I will describe a method for determining it later. The 1 or 2 Ω emitter bulk resistance of the 2N3904 oscillator transistor can be ignored if C3 and C4 are chosen so that the emitter current is less than 0.5 mA. If the oscillator resistance, R_{osc}, is the real part of Z_{in}, then at a temperature of 21°C (70°F), it is given by Equation 2.

$$R_{osc} = -\left(\frac{B\ I_e}{f^2\ C3\ C4}\right) \quad \text{[Eq 2]}$$

We will ignore the effect of the holder capacitance C_h because it is much smaller than the series combination of C3 and C4, and will be taken into account when the constant B is determined. The motional resistance, R_m, is equal to the magnitude of the oscillator resistance, R_{osc}, and so is proportional to the emitter current.

The imaginary part of Z_{in} is the series combination of C3 and C4. We will call this combination C_{osc}, which is given by Equation 3.

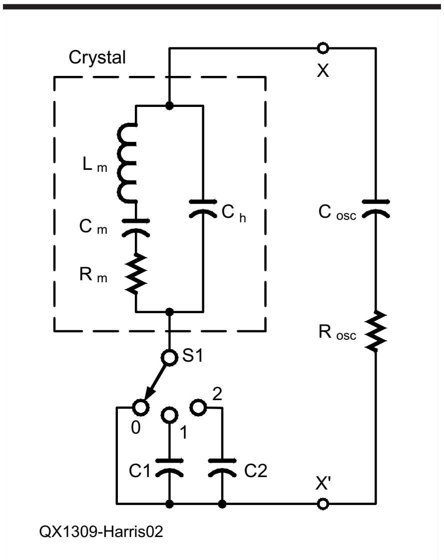


Figure.2 — Here is a diagram of an oscillator small signal equivalent circuit.

$$C_{osc} = \frac{C3\ C4}{C3 + C4} \quad \text{[Eq 3]}$$

The small signal equivalent circuit is shown in Figure 2. C_m, L_m and R_m are the motional parameters of the crystal. With the switch SW1 at position 0, the frequency of oscillation f_0 is given by Equation 4.

$$f_0 = \frac{1}{2\pi\sqrt{\dfrac{L_m\ C_m\left(C_h + C_{osc}\right)}{C_m + C_h + C_{osc}}}} \quad \text{[Eq 4]}$$

With the switch in position n, where n is 1 or 2, the frequency f_n is given by Equation 5.

$$f_n = \frac{1}{2\pi\sqrt{\dfrac{L_m\ C_m\left(C_h + \dfrac{C_n\ C_{osc}}{(C_n + C_{osc})}\right)}{C_m + C_h + \dfrac{C_n\ C_{osc}}{(C_n + C_{osc})}}}} \quad \text{[Eq 5]}$$

We can derive two equations for C_m by combining the equations for switch positions 1 and 0 (C_{m10}) and switch positions 2 and 0 (C_{m20}), eliminating L_m in the process.

$$C_{m10} = \frac{\left(\left(\dfrac{f_1}{f_0}\right)^2 - 1\right)}{\left(\dfrac{1}{\left(C_h + \dfrac{C1\ C_{osc}}{C1 + C_{osc}}\right)} - \left(\dfrac{f_1}{f_0}\right)^2 \dfrac{1}{C_h + C_{osc}}\right)} \quad \text{[Eq 6]}$$

$$C_{m20} = \frac{\left(\left(\dfrac{f_2}{f_0}\right)^2 - 1\right)}{\left(\dfrac{1}{\left(C_h + \dfrac{C2\ C_{osc}}{C2 + C_{osc}}\right)} - \left(\dfrac{f_2}{f_0}\right)^2 \dfrac{1}{C_h + C_{osc}}\right)} \quad \text{[Eq 7]}$$

The holder capacitance of the crystal will be the value of C_h that makes C_{m10} equal to C_{m20}, which will then be the motional capacitance C_m. I don't have an analytical solution for determining C_h, so I solve it numerically. For AT-cut crystals in standard or low profile HC49 packages, C_h will be within the range 1.5 to 6 pF. The microprocessor controller measures the three frequencies and then steps

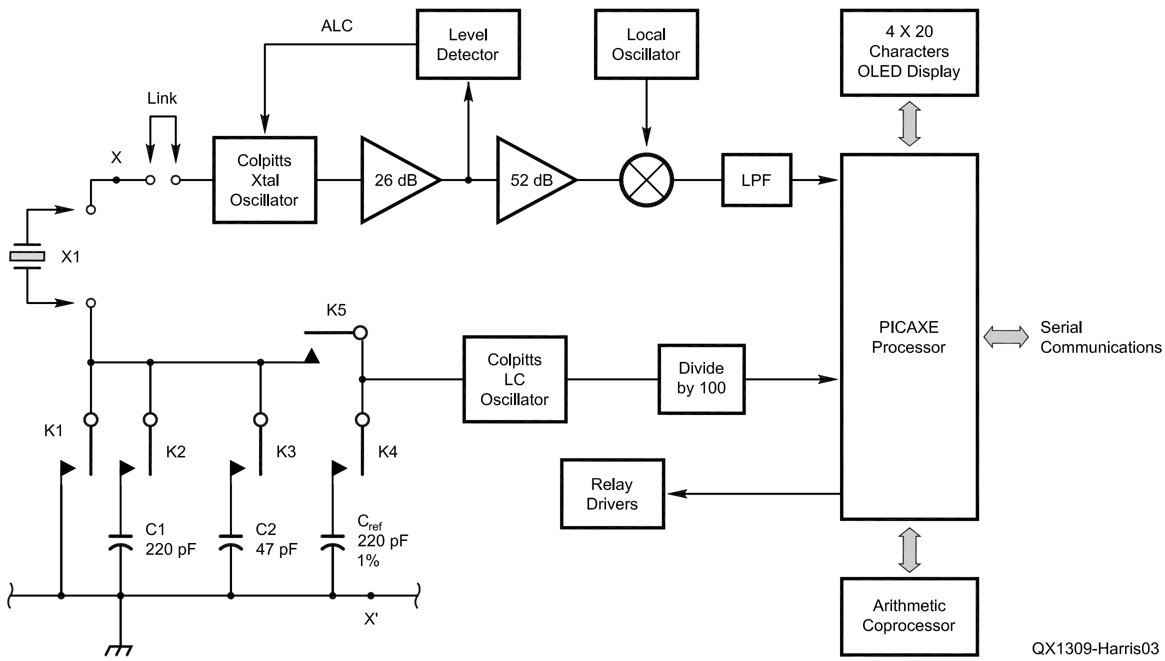


Figure 3 — The schematic/block diagram hybrid of the author's crystal measurement unit.

through the range of possible values for C_h in increments of 0.1 pF, calculating C_{m10} and C_{m20} at each step, until equality is found.

Once C_m and C_h have been determined, the series resonant frequency, f_s, of the crystal can be calculated from any of the three measurements of frequency. The simplest formula is when SW1 is in position 0.

$$f_s = f_0\left(1 - \frac{C_m}{2\left(C_h + C_{osc}\right)}\right) \quad \text{[Eq 8]}$$

The motional inductance L_m and Q can now be calculated by Equations 0 and 10.

$$L_m = \frac{1}{\left(2\pi f_s\right)^2 C_m} \quad \text{[Eq 9]}$$

$$Q = \frac{2\pi f_s L_m}{R_m} \quad \text{[Eq 10]}$$

We have now found all four of the component values and the Q of the equivalent circuit for the fundamental mode of operation.

A Practical Measuring Instrument

The block diagram of the measuring equipment is shown in Figure 3. Reed relays are used to switch capacitors in series with the crystal being measured. C1 (220 pF) and C2 (47 pF), both NP0 ceramic capacitors, could be measured before being fitted into the circuit, but relays K1, K2 and K3 add additional stray capacitance and the calculated motional and holder capacitances will be in error. My solution is to measure the capacitance in situ and this is the purpose of the Colpitts LC oscillator. Capacitance is measured relative to the reference capacitor C_{ref}. With the crystal oscillator turned off, and with K2 or K3 selected, three frequency measurements are made.

With relays K4 and K5 open, let the frequency be f_0. With K5 open and K4 closed let the frequency be f_{ref}. With K5 closed and K4 open let the frequency be f_x. K2 and K3 are open or closed as appropriate to the capacitor being measured. The unknown capacitance C_x can be calculated in terms of the ratio of the frequencies and the reference capacitor, C_{ref}.

$$C_x = C_{ref}\left(\frac{\left(\frac{f_0}{f_x}\right)^2 - 1}{\left(\frac{f_0}{f_{ref}}\right)^2 - 1}\right) \quad \text{[Eq 11]}$$

This method is similar to that described by Carver.[2] I selected an NP0 ceramic capacitor for C_{ref}, which I measured to be 220.0 pF using an Almost All Digital Electronics (AADE) LC meter. The inductor and the Colpitts capacitors are chosen so that f_0 is about 7 MHz, midway between the frequencies of the crystals to be measured. The microprocessor that I use cannot measure frequencies above a couple of hundred kilohertz, and so the frequency is divided by 100 before being counted. I found that the stray capacitance associated with the relays was about 7 pF, a significant addition to C1 and C2.

When crystals are being assessed, the LC Colpitts oscillator is turned off and K5 opened. To bring the frequency within the counting range of the processor, the amplified output of the crystal oscillator is mixed with an external oscillator and converted down to an intermediate frequency of 1 to 3 kHz. A CMOS exclusive-OR gate is used as a digital mixer and an RC low pass filter cleans up the output for counting. The crystal oscillator frequency is calculated from that of the external oscillator and the intermediate frequency.

The processor is a PICAXE 40X2, a 40 pin PIC with an integral "PICAXE basic" interpreter, and which is very easy to program. It controls the measurement sequence and also communicates by means of I^2C with a µM-FPU coprocessor that uses 32 bit float-

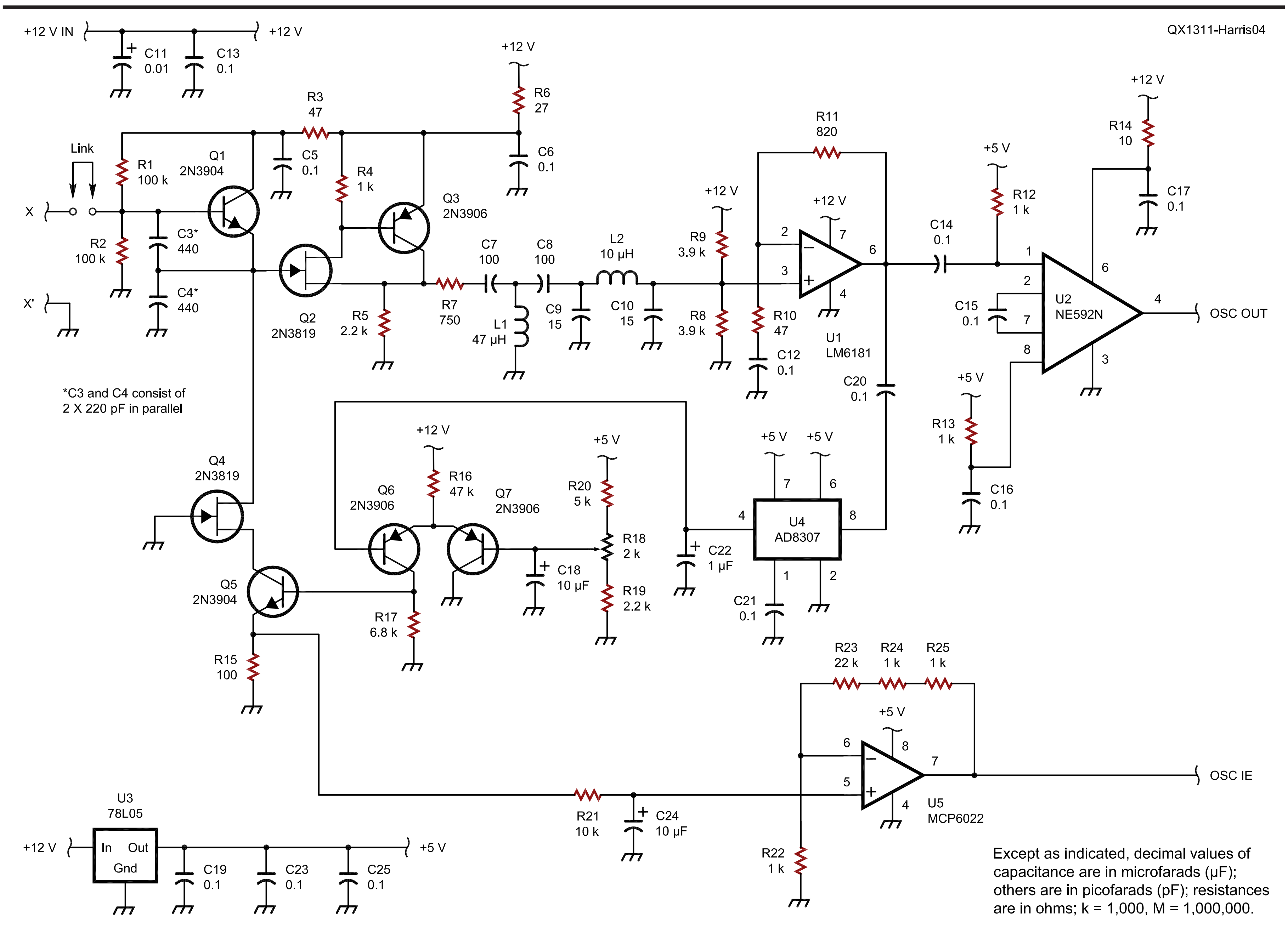


Figure 4 — This schematic shows the amplitude controlled Colpitts oscillator circuit used in the author's test system.

ing-point arithmetic. All of the calculations are made within the unit and the results are displayed on a 4 line by 20 character OLED display and are also sent to a computer running Microsoft *Excel*. Crystals can be measured at a rate of two or three a minute and the tabulated results sorted to group those with the similar values, such as motional inductance, series resonant frequency and motional resistance.

The level controlled crystal oscillator is at the heart of this unit and the schematic diagram is shown in Figure 4. The crystal and capacitor switching relays are connected to X-X'. Q1 with C3 and C4 form the Colpitts oscillator with the emitter current controlled by Q5. A unity gain buffer consisting of Q2 and Q3 drives a high pass filter, C7, L1 and C8, cascaded with a low pass filter,C9, L2 and C10, giving a pass band between 1 MHz and 20 MHz. U1 is a wideband current mode amplifier with a gain of approximately 20, which drives the ALC detector U4, an AD8307 logarithmic amplifier.

The long tailed pair Q6 and Q7compares the detected signal with a reference voltage set by R18 and the ALC loop is completed by means of Q5. A small voltage is developed across R15, amplified by U5, and then digitized and scaled by the processor to give the oscillator emitter current. Finally, U2 provides further amplification to drive the digital mixer.

The formula for the negative resistance of the oscillator has a constant, *B,* to correct for the deviation from the theoretical value of unity because the reduced current gain of Q1 at high frequencies lowers the input resistance of the transistor. We can make a reasonable estimate of *B* by first using the equipment to measure the motional resistance of a number of crystals and choosing the one with the lowest value. The link in series with the crystal is removed and several fixed resistors, for example 10, 15, 22 Ω, fitted in turn and measurements made again. The motional resistance now includes a known fixed resistor. The constant, *B,* is the best fit value that makes the increase in the measured resistances to be the same as the resistors used. For the 2N3904, I found that *B* was 1.20 for 5 MHz crystals and 1.35 for 10 MHz crystals.

The oscillator unit uses "Manhattan" style construction and is shown in Photo A. The crystal oscillator, capacitor switching relays and ALC circuit are on the left hand side. The lead photo on page 3 shows the assembled measurement unit and controller. The external oscillator, a homemade DDS signal generator, is not shown in the photograph. The calculated values of the motional capacitance and inductance are displayed to six significant digits, despite being only accurate

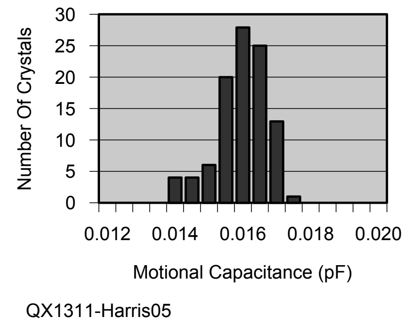


Figure 5 — This graph shows the spread of motional capacitance, C_m, for a batch of 100 crystals.

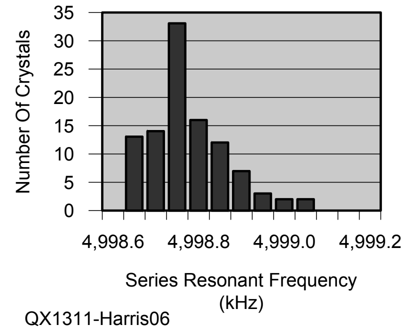


Figure. 6 — This graph shows the spread of series resonant frequencies, f_s, for the same 100 crystals.

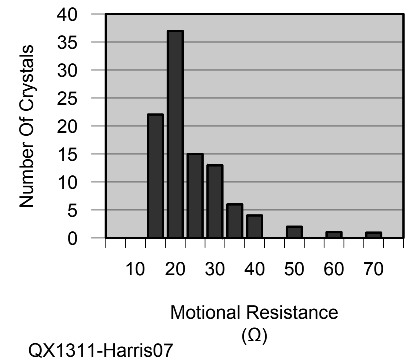


Figure. 7 — This graph shows the spread of motional resistance, R_m, for the batch of 100 crystals.

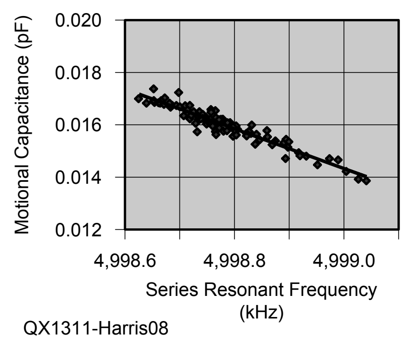


Figure. 8 — Here is a plot of the motional capacitance, C_m, versus series resonant frequency, f_s, for the batch of crystals.

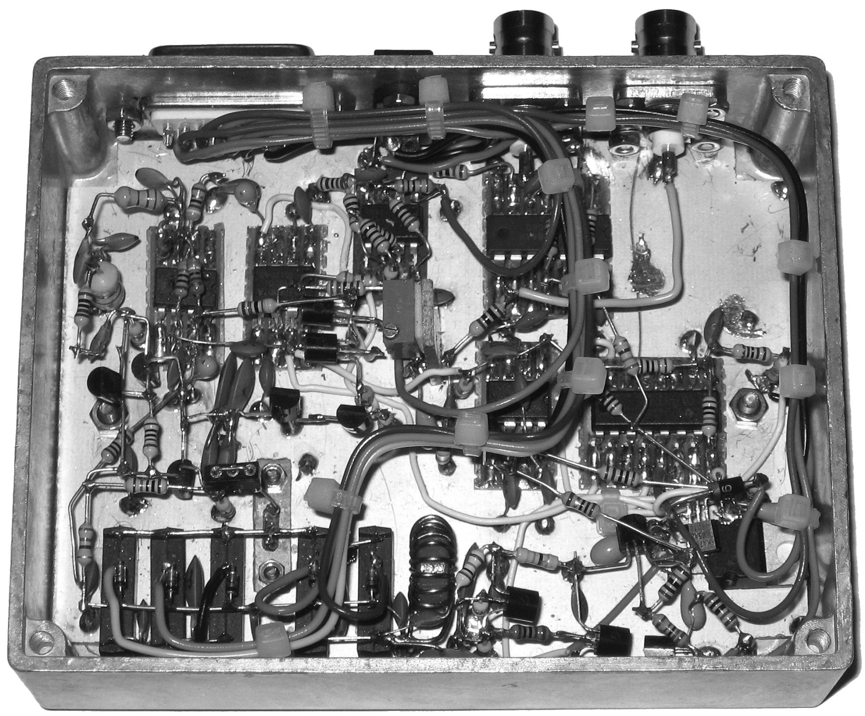


Photo A

to about 1%, because otherwise the center frequency of a filter design using values to, say, 3 significant figures could be in error by tens of kilohertz.

Some Results

I have measured several hundreds of crystals. I will describe the results of one batch consisting of 100 crystals purchased from a major UK component distributor. The nominal frequency was 5 MHz, with a load capacitance of 30 pF and a tolerance of 30 ppm. These were housed in HC49/U packages and cost less than 30 cents each.

Before we look at the spread of these motional parameters, it is worth considering what the selection priorities should be for crystals that are to be used in a filter. My personal view has been that the motional inductance is the first consideration, and the crystals should be selected to have as small a spread as possible. From this sub-set those with similar series resonant frequencies can be selected. Most crystals in a filter will be in series with capacitors to give the correct mesh frequencies and if necessary these capacitors can be adjusted to take into account small differences in the series resonant frequencies of the crystals. Finally, from this much smaller sub-set, those with similar, and small, motional resistances can be selected.

I numbered the crystals individually and measured the motional parameters of each one. The time taken to measure 100 crystals was about 40 minutes. The results were displayed in tabular form in an *Excel* spread sheet. The important motional parameters were sorted into various "bins," 0.5 *f* F wide for the motional capacitance, 50 Hz wide for the series resonant frequency and 5 Ω wide for the motional resistance. The distributions of these motional parameters are shown in Figures 5, 6 and 7. More than 25% of the crystals fell within one "bin" on each of the graphs but, unfortunately, there is no guarantee that the same crystals were in each of these bins.

Inexpensive crystals are manufactured for use in oscillators. Provided that the frequency with the specified parallel load capacitance is within the stated tolerance, then the motional capacitance and inductance can in principle take any value. The series resonant frequency is given by Equation 12.

$$f_s = f_p\left(1 - \frac{C_m}{2\left(C_h + C_p\right)}\right) \qquad \text{[Eq 12]}$$

where f_p is the specified resonant frequency with the specified parallel load capacitance, C_p. So if C_m can take a range of values, then so can f_s. If motional capacitances are plotted against series resonant frequencies, however, then the result is a straight line with a slope given by Equation 13.

$$\frac{d\,f_s}{d\,C_m} = \frac{-f_p}{2\left(C_h + C_p\right)} \qquad \text{[Eq 13]}$$

where f_p is the nominal frequency with the load capacitance C_p. The slope is –73.5 Hz / *f* F for 5 MHz crystals with a specified load capacitance of 30 pF and holder capacitance of 4.0 pF. The motional capacitances and the series resonant frequencies of the crystals are shown in Figure 8, along with the least squares trend line, which has a slope of –76.1 Hz / *f* F, close to the calculated figure. The motional inductance has a similar relationship, although with a positive slope.

There is a fortunate consequence of this relationship. Crystals selected for a small spread of series resonance will also exhibit a small spread of both motional inductance and capacitance. Selecting crystals based solely on similar series resonant frequencies is a viable method. Furthermore, crystals from different manufacturers with the same nominal frequency, load and holder capacitances can be mixed and selected on this basis.

I constructed a 5 MHz pre-distorted linear phase filter designed from the tables of *k* and *q* values in Zverev using the average motional parameters of six crystals selected from this batch.[3] I added L-match circuits to the input and output of the filter to match to the 50 Ω terminations of the signal generator and detector. The plotted amplitude and phase responses overlaid those predicted by *SPICE* over a 60 dB amplitude range (the limit of the test equipment) and a phase range of 900°.

Accuracy

I have found that the measured motional capacitance and inductance of other crystals agreed with another method that I have described to within better than 1%.[4] Ultimately, all measurements are related to the accuracy of my LC meter, which is specified to be 1%.

As a check, I measured the series resonant frequencies of a sample of 30 crystals by connecting the output of a DDS signal generator to each crystal in turn through a series resistor and adjusting the frequency for minimum voltage across the crystal. I then re-measured the same crystals using the Colpitts oscillator method so that I could make a comparison at the same room temperature. The series resonant frequency using the direct measurement was on average 13 Hz higher than that found using the oscillator method. This discrepancy is probably due to the additional inductance of the wiring through the relays, which I estimate to be 60 nH.

I also calculated the motional resistances of the sample from the source voltage, the voltage across the crystals and series resistor value. The motional resistances agreed quite well with the oscillator method, being on average 6% higher. I have found that the motional resistance of some crystals, however, is not constant and varies with crystal current so that such comparisons are not entirely reliable.[5]

The holder capacitance was within 0.1 to 0.4 pF of that measured using an LC meter for 5 and 10 MHz crystals in both standard and low profile packages.

Conclusion

A Colpitts oscillator has been described that can be used as the basis of an instrument for measuring all of the motional parameters of quartz crystals. When controlled by a microprocessor, the results can be sent to a spreadsheet and displayed in tabular form to enable crystals with similar properties to be selected. With the present equipment, up to 150 crystals an hour can be measured.

Richard Harris was licensed as G3OTK in 1961. He received Bachelor and Master Degrees in Electrical Engineering from the University of Bath in the UK. He is a Member of the Institution of Engineering and Technology and a Chartered Engineer. Although he has spent much of his professional life undertaking electronic design, for the last ten years he has been responsible for Quality Assurance, Health and Safety and Environmental Management. He is a member of the Itchen Valley Amateur Radio Club.

Notes

[1]Wes Hayward, W7ZOI, Rick Campbell, KK7B and Bob Larkin, W7PUA, Experimental Methods in RF Design, published by the ARRL, 2009, ISBN: 978-087259-923-9; ARRL Publication Order No. 9239, $49.95. ARRL publications are available from your local ARRL dealer or from the ARRL Bookstore. Telephone toll free in the US: 888-277-5289, or call 860-594-0355, fax 860-594-0303; www.arrl.org/shop; pubsales@arrl.org.

[2]B. Carver, "The LC Tester," *Communications Quarterly,* Winter 1993, pp 19 – 27.

[3]A. I. Zverev, *Handbook of Filter Synthesis*, Wiley-Interscience.

[4]Richard J. Harris, G3OTK, "Crystal Bridge — A Balanced Bridge for Measuring Quartz Crystal Parameters", *Rad Com*, September 2011, pp 50 – 52.

[5]Richard J. Harris, G3OTK, "The Drive Level Sensitivity of Quartz Crystals," *QEX* Jan/Feb 2013 pp 14 – 21.