NØAX

# HANDS-ON RADIO

## Experiment #75 — Series to Parallel Conversion

Radio hardware designers, like magicians, have a repertoire of interesting and useful tricks. As with magic, the simple and familiar tricks are often the most useful because they can be used in many ways and many situations. So it is with circuit transformations.

### Equivalent Circuits

Transformations are really all about *equivalent circuits*, the ability to replace one section of a circuit with a different set of components without changing any of the voltages and currents. Most equivalent circuits are *one-port* equivalents in that they are applied at a pair of terminals called a *port*.

Let's start with a very simple equivalent circuit. Suppose you have a circuit composed of a single 12 Ω resistor. It's sealed up in an opaque box and you connect to it through a pair of terminals. This pair of terminals is the port. What equivalent circuits can you think of that would appear to be a 12 Ω resistor from outside the box? I'm sure you can think up several: twelve 1 Ω resistors in series, two 24 Ω resistors in parallel, a 6 Ω resistor in series with the parallel combination of a 24 Ω and 8 Ω resistor, etc.

There are an infinite number of resistor combinations that produce a 12 Ω resistance between those terminals. These are all equivalent circuits for the single 12 Ω resistor. Using an ohmmeter, you could never know the difference between any of the combinations.

### Why Equivalent Circuits?

Why are equivalent circuits so common in electronics? In a word, simplification. General class licensees learned to do simple equivalents to pass the exam — converting series and parallel combinations of resistance, capacitance and inductance into a single equivalent component that acted just the same as the combination of components they replaced. Why do calculations for multiple components, when a single equivalent component will represent the combination just fine?

Another simplification is that components, sections of circuits and sometimes even entire pieces of equipment can be replaced with a simple equivalent circuit with which it is easier to model or calculate behavior. Another example that comes quickly to mind is an RF signal generator. For all intents and purposes, to the external world it appears to be an ac voltage source in series with a 50 Ω resistor. That's oversimplifying the generator, but in most cases that simple model will do just fine. (The Thevenin and Norton equivalent circuits are good examples of this type of simplification and are discussed in Hands-On Experiment #32.[1])

Equivalent circuits can also change the form of a circuit so that it is easier to analyze or use in a design. An equivalent circuit might provide a better way to describe the behavior of the circuit. At any rate, having a set of mathematical tools to change one type of circuit into another is a valuable skill.

### AC Equivalents

What about ac circuits with capacitors and inductors — do they have equivalents, too? Yes, but because the behavior of those circuits changes with frequency, the equivalent circuit is generally only an exact replacement at one frequency.

A simple component (R, L or C) is more complex than you might think. For example, an inductor has loss associated with the resistance of the wire. A capacitor's dielectric dissipates some of the stored energy as heat. A resistor's leads act like small inductors. All of these *parasitic* effects can be significant in certain applications. Consequently, test instruments that measure component value also measure the parasitic values and can provide the measurement as one of several different equivalent circuits.

The most common equivalents for measurements of capacitors and inductors are parallel R and series R, respectively, as shown in Figure 1. For capacitors, a parallel resistor most closely represents the effects of loss in the dielectric. For inductors, losses in the wire are best represented by a series resistance. Resistors are measured in the same way, except that R is the primary component and series L or parallel C are the parasitic effects.

[1]Previous Hands-On Radio columns are available to ARRL members at **www.arrl.org/tis/info/HTML/Hands-On-Radio**.

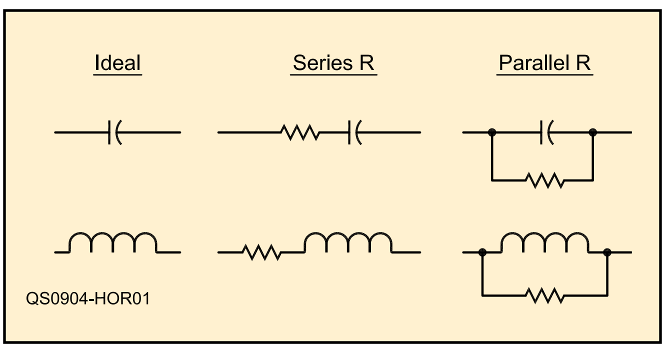


**Figure 1 — Losses in non-ideal capacitors and inductors are modeled as series and parallel resistances. Parasitic capacitance and inductance in resistors are modeled as parallel and series components.**

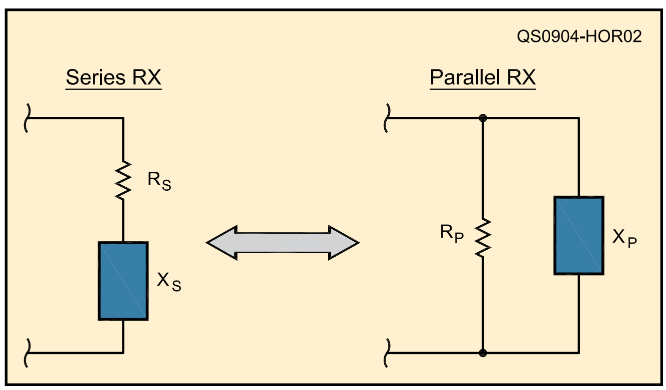


**Figure 2 — Series circuits containing resistance and reactance can be transformed into parallel circuits (and vice versa) to simplify circuits and aid in design tasks.**

H. Ward Silver, NØAX ♦ PO Box 927, Vashon, WA 98070 ♦ n0ax@arrl.org

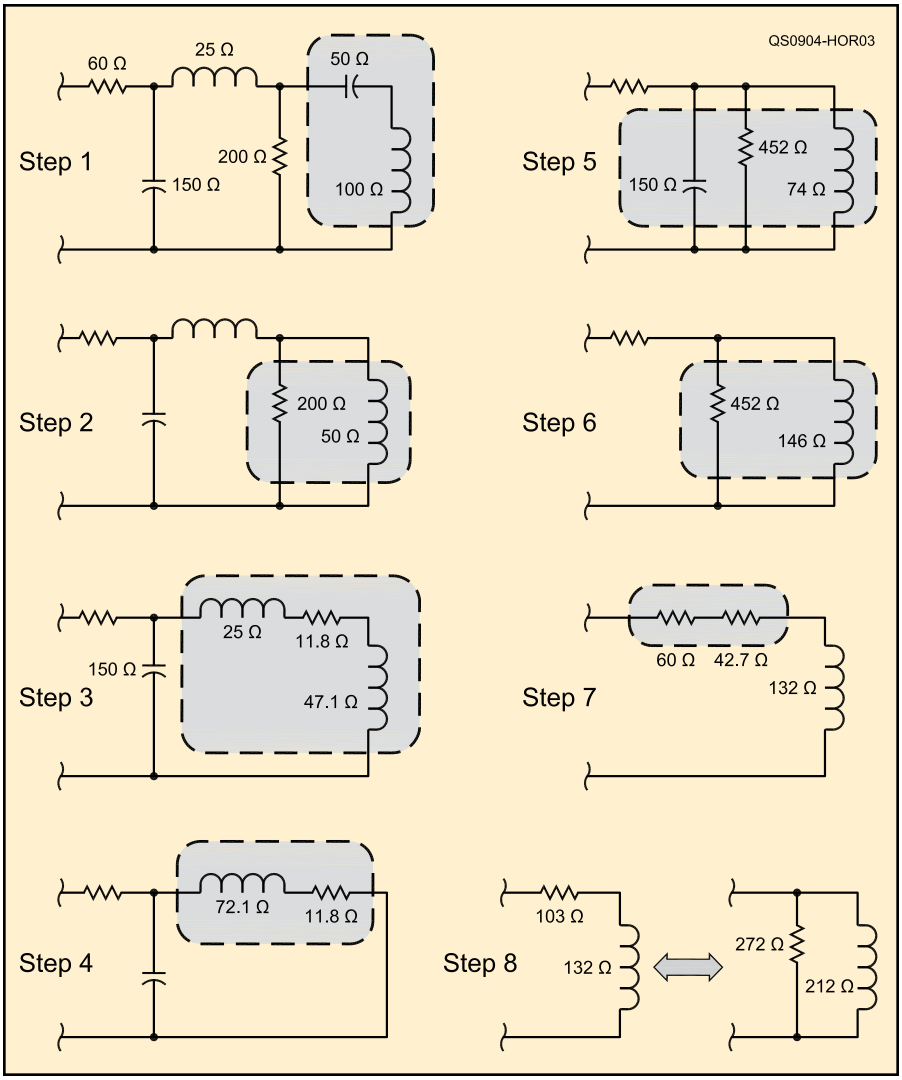


**Figure 3 — A series of step-by-step transformations simplifies the complex circuit at the upper left into a simple series-RX or parallel-RX equivalent.**

## Series to Parallel Transformation

You already know how to work with combinations of series and parallel resistances to turn them into their simplified equivalents. Parallel and series reactances can be combined, as well. Yet at some point, it is often useful to convert between a series R-X circuit and its equivalent parallel R-X circuit as shown in Figure 2. Two circuits are ac equivalents if the same current flows with the same phase angle when a given voltage of the same frequency is applied to both circuits. (Remember that the equivalent is only an exact replacement at a specific frequency.)

When you have reduced the circuit to a single resistance and a single reactance, either in parallel or series, use the following formulas to convert between series and parallel circuits. If the circuit can be drawn as R in series with X, it can be converted to R in parallel with X as follows:

$$R_P = \frac{R_S^2 + X_S^2}{R_S} \qquad X_P = \frac{R_S^2 + X_S^2}{X_S} \qquad \text{[Eq 1]}$$

If the circuit can be drawn as R in parallel with X, it can be converted to R in series with X as follows:

$$R_S = \frac{R_P X_P^2}{R_P^2 + X_P^2} \qquad X_S = \frac{R_P^2 X_P}{R_P^2 + X_P^2} \qquad \text{[Eq 2]}$$

These formulas assume that the absolute value of reactance is used, so that X is always a positive number. The reactance will be of the same type (capacitive or inductive) before and after the series-parallel transformation.

Let's start with two simple examples. In the circuit of Figure 2A (series RX), let $X_S = -50\ \Omega$ (capacitive) and $R_S = 50\ \Omega$. Use formula set 1 to convert to the parallel combination of $X_P = -100\ \Omega$ (still capacitive) and $R_P = 100\ \Omega$. In the circuit of Figure 2B (parallel RX), work the problem backwards, using formula set 2.

## Step by Step

Figure 3 shows a more challenging problem — reducing the circuit shown at Step 1 to its series or parallel equivalent. This takes eight steps of simplification, the component values to be transformed or combined circled by a dashed line. The sequence shows the circuits getting progressively simpler until the only circuits left are the series RX and parallel RX equivalents. (Values shown are the results of the required operation.)

Wherever there are reactances in series, combine them by subtracting $X_C$ from $X_L$. One more formula is required. Combine parallel reactances by using the formula

$$X_{total} = \frac{-X_L X_C}{X_L - X_C} \qquad \text{[Eq 3]}$$

Again, use only the magnitudes of the reactances. If the result is negative, $X_{total}$ is capacitive and if positive, $X_{total}$ is inductive.

Start at the far right of the circuit, combining the two series reactances, leaving 50 Ω of inductive reactance.

- Transform parallel RX into series RX with formula set 2.
- Combine the series reactances, leaving 72.1 Ω of inductive reactance.
- Use formula set 1 to transform series RX into parallel RX.
- Use formula 3 to combine the parallel reactances.
- Transform parallel RX to series RX using formula set 2.
- Combine the series resistances, leaving 132 Ω of inductive reactance in series with 103 Ω of resistance.
- Use formula set 1 to transform series RX to parallel RX.

## Recommended Reading

The "Electrical Fundamentals" chapter of *The ARRL Handbook* sections on reactance and impedance go into more detail about transformations.[2] Most circuit texts will cover the "Wye-Delta" or Y-Δ transformation that makes an appearance in radio circuits to change T networks into π networks.

## Next Month

The lowly diode, simplest of all semiconductor devices? Not quite so simple, as we'll see next month as you compare several types and find out how they're used.

[2] *The ARRL Handbook for Radio Communications*, 2009 Edition. Available from your ARRL dealer or the ARRL Bookstore, ARRL order no. 0261 (Hardcover 0292). Telephone 860-594-0355, or toll-free in the US 888-277-5289; **www.arrl.org/shop/; pubsales@arrl.org**.